Sugar and Spice

Stir in Problem Solving into Your Grade 7 & 8 Mathematics Classroom

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A Balanced Diet!

Problem Solving in the Mathematics Classroom

In our adult world, we face challenges and problems every day! Everything from balancing the home budget, to preparing meals, fixing the leaky faucet and predicting when to buy gas at its lowest price! As educators, if we can help our students to perceive problems as challenges rather than obstacles that stand in their way, then we will make a profound difference in their lives.

As catholic educators, we share a vision of what the Catholic graduate learner should look like. We believe that each student should strive to be 'effective communicators' (CGE 2), both in literacy and in numeracy. Expectations from the Ontario Curriculum stress verbs such as describe, explain, model, relate, demonstrate, and justify. When students are asked to communicate, explain and reflect back on their thought processes then they learn from the retelling and relating not just the 'recalling' of information given to them.

We encourage our students to be 'reflective, creative and holistic thinkers' (CGE 3). Looking at a problem and realizing that there may be many solutions to the same problem adds depth and appreciation of the problem's complexity. Students learn to continue to search for answers even though one solution has been found. Looking at the whole problem and its
context, rather than teaching skills in isolation, fosters real-world connections and relevance for students. No longer will we hear, “When are we ever going to use this?”

Most importantly, we want our students to be ‘self-directed, responsible lifelong learners’ (CGE 4). Educational changes in the last decade and research based on how we learn have shown students to be more than passive receptacles of learning to be filled up with rote knowledge. For real and meaningful learning to take place, students have to make sense of their world by connecting prior knowledge to new experiences. “Learning occurs not from the outside in, but from the inside out.” (Billmeyer and Barton. Teaching Reading in the Content Areas: If Not Me, Then Who?) To value and showcase learning as a continual and dynamic cycle allows our students to gain independence and to value the skills needed for a lifetime of learning.

In this new millennium, where information and technology grows at an exponential rate, the ‘old competencies’ of reading, writing and arithmetic are essential but a more varied set of skills is needed to face the complexity of rapid change. Students need experiences in manipulating thoughts and abstract concepts to find patterns and to see relationships of parts to the whole. To be successful students need to question, conjecture, investigate, explore, test and apply their understanding. And, they need to do this in a spirit of community as ‘collaborative contributors’ (CGE 5) where there is an appreciation for the strengths and weaknesses of each other as God’s children. Through discussion, questions and dialogue, students share ideas and grow in their depth of understanding.

Problem-based or inquiry-based strategies value these learning conditions, forcing a shift in roles for both the student and the educator.
Responsibility for making connections, establishing their own understanding and validating their thought processes is placed in the hands of the students. The educator is no longer the holder and giver of knowledge but takes on a new role as facilitator. Shaping the experiences, engaging students in authentic learning, questioning, challenging, clarifying and helping to validate thinking becomes the role of the teacher.

Educators and students will come to see Mathematics not as a discipline of prescribed skills but as a doing and interactive process.

“When students learn mathematics, they do more than master basic skills; they acquire a concise and powerful means of communication”

*Ontario Curriculum 1997*

“When better learning will not come from finding better ways for the teacher to instruct, but from giving the learner better opportunities to construct”

Billmeyer and Barton.

*Teaching Reading in Content Areas: If Not Me, Then Who?*

Long ago, Confucius said, “Tell me and I’ll forget; show me and I’ll remember; involve me and I’ll understand!” Mathematics is seen as a way of thinking, communicating and making sense of this world through interaction, inquiry and problem solving!
Daily Specials!

Two Essential Themes

There are really only two themes needed in a Mathematics problem-based classroom: ‘Math is a study of patterns’ and ‘Math is all around us’! If you can introduce problem solving and model the acquisition of concepts as a continual search for patterns to organize information and make understanding easier, then students become adept at finding their own patterns. Explain that as human beings we are always looking for patterns to make sense of the world. Encourage them to act as detectives, looking for clues to solving the puzzle! Recognizing a pattern makes an impossible challenge look easy the second time around!

The challenge ‘Petals Around the Rose’ forces students to look for a distinctive pattern. Once they find it, they reflect back and wonder why the challenge was so hard in the beginning! (See pg 9) Find the solution to “Petals Around the Rose’ in ‘Secret Ingredients: Solutions and Comments’. Pg. 86.

Focus on the saying ‘Math is a study of patterns’ every day to highlight the importance of seeking patterns and organizing new information into well-developed patterns of understanding.

With the theme ‘Math is all around us!’ who needs a textbook! By using real-world problems and situations, students will become exposed to the world around them and better able to make connections. If you are teaching a concept in your classroom and you can’t find a real-world connection, then perhaps you should wonder, “Why am I teaching this?”
At the beginning of the year, brainstorm a list of where ‘Math is all around us!’ with your students. Keep the list handy as you cluster expectations and develop culminating tasks. Listed below are examples of Math in everyday life.

<table>
<thead>
<tr>
<th>Math is in...</th>
<th>Strand</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Architecture</td>
<td>Geometry</td>
</tr>
<tr>
<td>• Cooking</td>
<td>Number Sense; fractions/Science: capacity</td>
</tr>
<tr>
<td>• Construction</td>
<td>Geometry: 3D and plane geometry</td>
</tr>
<tr>
<td>• Shopping</td>
<td>Number sense: percents, decimals</td>
</tr>
<tr>
<td>• Stock market</td>
<td>Number Sense: integers</td>
</tr>
<tr>
<td>• Renovations</td>
<td>Measurement: Area, perimeter</td>
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<td>• Product design</td>
<td>Measurement: volume</td>
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<tr>
<td>• Quilting</td>
<td>Measurement; Geometry</td>
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<tr>
<td>• Kitchen tiles; interlock</td>
<td>Geometry: Tessellations</td>
</tr>
<tr>
<td>• Sports</td>
<td>Data management</td>
</tr>
<tr>
<td>• Weather</td>
<td>Probability</td>
</tr>
<tr>
<td>• Business</td>
<td>Patterning and Algebra; Number Sense</td>
</tr>
</tbody>
</table>
Petals Around the Rose (Source Unknown)

- Five dice are thrown and using a specific pattern, a 'total' is found for each roll.
- The 'total' will always be even.
- The name of the activity, 'Petals Around the Rose', is very important in understanding the pattern.

First roll:   Total: 10

Second roll:   Total: 8

Third roll:  Total: 4

Maximum roll:  Total: 20

Zero roll:
Snacks: Bet you can’t serve one!
Students will want more!

Making Math Magical!

Math should be presented as fun and intriguing! Take the role of a magician as you present some of these activities! Use them to 'hook' your students. Have them investigate the problem. Can they come up with their own problems to astound the class?

Challenge: Can you walk through a piece of paper?

Challenge students to manipulate and cut a piece of paper so that the elongated 'loop' can be stretched out and someone can easily walk through it! No staples, tape or glue can be used! The answer for 'Walk through Paper' is found in 'Secret Ingredients: Solutions and Comments', pg. 86.

Happy Birthday! Here’s a trick that will astound your students.

Ask students to write down the month he/she was born.
(Example: Feb - write down 2)

Multiply this number by 10

Add 10

Multiply the result by 10

Add your age in years

Subtract 100

Notice: 2 represents February and 12 is the student’s age! (Adapted from *Mathemagic*. Don Fraser.)
**Keep on Subtracting**: Using the template, start with any four numbers and 'keep on subtracting'. Look at the example below. Have each student try a different set of four numbers. The record is 10 sets of subtractions before zeros were reached! The template for 'Keep on Subtracting' is found in 'Sharing Recipes', 67.

![Number Diagram](image-url)
To develop a problem-based or inquiry-based classroom requires one essential element that needs to be fostered and encouraged on a daily basis. Students need to feel that they can take risks in class without the threat of teasing or having their ideas dismissed as trivial.

The foundation of problem solving is based on questions, such as, 'what if?' and 'I wonder if this would work?' If students are reluctant to offer their opinions, then the process of considering possible solutions grinds to a halt. Through discussion, modeling and well-chosen activities, students will come to appreciate the importance of respecting the contributions and rights of others (CGE 5e). This may be one of the most valuable lessons they will learn in developing a sense of community.

'Working effectively as an interdependent team member' (CGE 5a) doesn’t just happen! In a grade 7 and 8 classroom, when peer pressure mounts and social acceptance is of the utmost importance, working together to reach a common goal has to be nurtured and molded.

A risk-taking environment has to be developed throughout the year. Consider these suggestions to assist you in constructing a classroom community where every student feels safe and valued.

- Involve students in creating classroom expectations and routines.
Plan activities that focus on demonstrating social skills, such as, working cooperatively, listening and disagreeing in an appropriate way.

Model and reinforce sentences starters that may help students ask clarifying questions, ask for opinions or explanations.

Role play how to resolve a conflict. Demonstrate how to reach consensus.

Consider a key word as a cue for any student, and the teacher, to stop and rephrase a comment that someone in the community may not feel is appropriate. Calling out the word 'reset' works well in reminding individuals to start their conversation again using a more acceptable tone or content.

Conference with individual students who have difficulty following classroom rules and devise strategies and cueing techniques to help these students.

Assign classroom responsibilities to students. If working with manipulatives, make sure students understand the value of taking care of materials.

Discuss how groups need to have a variety of skills sets within the group. Assign roles and responsibilities to group members until they are able to distinguish appropriate roles themselves.

Choose classroom groupings carefully with sensitivity to ability levels, gender and social issues. Vary groupings often to ensure that everyone in the class works with each member of the class throughout the year.

Use a variety of groupings: large groups, in fours, in threes or in pairs.

Encourage students to help each other by explaining their thinking, not by just giving each others the answers.
• Offer a variety of ways to demonstrate learning to accommodate learning styles and multiple intelligences.

• Allow students to have input into assessment criteria. Encourage self-assessment and peer assessment.

• Allow time for students to debrief after a major project. Have them discuss things that went well and how they might improve group weaknesses.

• Celebrate successes! Nothing encourages risk-taking like feeling success and achievement! Highlight student solutions and demonstrate different solutions of the same problem.
Teaching developmentally, from the concrete to the abstract is an essential component of an inquiry-based mathematics program. Students are encouraged to explain their solutions either verbally or through words and visual representations. Not every student will be ready in grades 7 and 8 to use the abstract symbols of mathematics. Having concrete materials available will be an important alternative for students to explain their reasoning.

Many students may have ‘missed’ the concrete representations of certain concepts and may need to review or make connections with prerequisite knowledge. It is very difficult for students in grade 7 to grasp the concept of an exponent as repeated multiplication if they haven’t really understood that multiplication is repeated addition! Assess their understanding of exponents by asking students to represent $3^3$. Many students may be able to use expanded notation and write $3^3 = 3 \times 3 \times 3$ but how many will be able to translate this equation into the written statement: three groups of, three groups of three and then illustrate?

Many students who are visual or kinesthetic learners will need manipulatives to assist in solving a problem or to justify and defend their
solution. During problem solving activities (and to support more traditional instructional strategies), students should have access to:

• calculators; rulers and metre sticks;
• bingo chips or tokens;
• number tiles (sets of digits, 0 to 9 to explore number relationships);
• multi-link cubes (to investigate fractions and measurement concepts);
• algetiles (to represent integers and algebraic equations concretely);
• geoboards (to explore geometry concepts);
• mira (to be used in transformational geometry to investigate symmetry, translations and reflections);
• dice, cards, and spinners (to conduct experiments in probability);
• fraction circles or strips (to represent fractions);
• cuisenaire rods (to be used in perimeter and area investigations);
• tangrams and pentominoes (to investigate spatial relationships, fractions and measurement concepts of perimeter and area):
• pattern blocks (to investigate patterns in algebra, geometry and tessellations);
• toothpicks (to represent algebraic patterns);
• blackline master copies of centimeter dot paper and isometric dot paper (to assist in drawing 3D structures);
• 3D geometric shapes;
• compasses and protractors (for geometry and constructions);
• AND LOTS OF SCRAP PAPER!

Students should have access to technology as well. The use of
spreadsheets to organize and graph data is a real-life application which is relevant to students. Introducing the basic functions of graphic calculators in grades 7 and 8 is a prerequisite for investigations in high school. Software programs, such as, Geometer Sketchpad, make investigations of concepts in geometry student-friendly!
“My students can’t do problem solving because they can’t read the problems!” This is a comment frequently uttered by grade 7 and 8 teachers. Whether you are a home-base teacher or a subject-specific math teacher, the issue of literacy has blossomed into a focus of concern. Research based on balanced literacy programs strongly suggests that reading skills have to be taught consistently in all content areas.

Literacy in math means “the ability to make sense of everything on the page – whether the page is a worksheet, a spreadsheet, an overhead transparency, a computer screen, a mathematics textbook or journal – in other words, any resource that students might use to learn and apply mathematics.” (Barton and Heidema. Teaching Reading in Mathematics.) Readers still have to use the same skills as in other subjects, such as, decoding the text, looking at the structure of text for hints of meaning, monitoring and reflecting on how effectively they are reading, and making connections with prior knowledge to new information. Struggling readers experience other challenges as well.

Decoding the ‘language of Math’ presents readers with the need to develop a new dimension of understanding. In Math, symbols are linked to abstract concepts that have to be recalled before any attempt can be made to solve a problem. Even the structure of math text can be a challenge! Looking at graphs and number lines may mean that information has to be ‘read’ from right to left or up and down.
'Translating' math terms to words or words to mathematical notation can be a challenge for many students. Consider terms in the English language that don't have the same connotation in math! The term 'difference' in Math implies the operation of subtraction but in the English language it implies a comparison. A simple question, answered without much difficulty, can become a problem when the student is faced with communicating a solution. 'How many groups of six in twenty-four?' can be represented mathematically in several ways – \( \frac{24}{6} \) or \( 24 \div 6 \). (Adapted from Barton and Heidema. Teaching Reading in Mathematics.)

Other obstacles exist for students in reading text. Many terms or concepts are entirely new to students. There is no prior knowledge to help draw meaning. A student may never have encountered the term 'exponent' before grade 7. Conversely, many concepts are spiraled and students should have an understanding of the concept from earlier grades. This may be problematic if the teacher does not assess prior knowledge and makes the assumption that students have the prerequisite knowledge.

Reading strategies introduced in language instruction can easily be transferred to the math class. Before beginning a problem set based on new concepts engage students in discussions related to the concepts. Use the theme 'math is all around us' as a starting point. Activate prior knowledge by using graphic organizers, such as, a concept map*, a semantic mapping chart* or a Frayer Model*. A concept rating chart* allows you to survey prior knowledge. To help in vocabulary development, have students preview a chapter in the text to look for terminology or symbols that are new to them. A word wall can display word sorts*, number sorts* or vocabulary with definitions and visual representations.
To assist students as they tackle a new problem, consider 'paired-reading' partnerships. One student can read while the partner summarizes information or asks clarifying questions. Struggling readers will benefit from reciprocal teaching. In conference with the students, the teacher models how to read a problem and verbalizes the thought processes while reading. Teacher and student take turns looking at different problems and verbalizing the decoding of the content. The use of graphic organizers, such as, a K-N-W-S* chart or a Five Step Process Chart* could be used with this strategy.

Another strategy to be used while reading is to have students, consistently, using a coding strategy to mark the problem with simple symbols that reflect their understanding.

<table>
<thead>
<tr>
<th>Coding Strategy:</th>
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</thead>
<tbody>
<tr>
<td>** I understand this term and know how to use it.</td>
</tr>
<tr>
<td>+ Information I will need to solve the problem</td>
</tr>
<tr>
<td>! Important information!</td>
</tr>
<tr>
<td>?? I don’t understand this term; I don’t know if this is important.</td>
</tr>
</tbody>
</table>

The use of graphic organizers at any stage of reading or working through a problem is vital. Graphic organizers provide a framework for organizing facts and concepts using a visual representational. Encourage the use of these tools by explaining their importance, modeling how to use a specific chart by using familiar concepts then model its use using new information. Encourage students to develop their own organizers or adapt
existing models. Software programs, like Inspiration, allow students to create their own graphic organizers quickly and easily!

Give students lots of practice at translating symbols into words and words into symbols. Initially, brainstorm a list of words that suggest an operation or concept, such as, total, sum, difference, product, less than and greater than. Create a bulletin board that matches the corresponding word and symbol. Add to the wall each time a student finds a new word. Try an activity where you give each student an equation. Each student writes three stories that would match the equation. Students exchange their stories and then translate the stories back into math symbols. Start with simple equations and progress to more difficult equations.

**Examples of Reading Strategies** (*Templates can be found in 'Sharing Recipes*)

*Concept definition mapping* refers to a graphic organizer that helps students understand the characteristics, qualities or properties of a term or concept. Students describe the concept in their own words, give examples of what it is, make comparisons, and list properties. (See 'Sharing Recipes', pg. 68.)

<table>
<thead>
<tr>
<th>Concept: Percent</th>
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</thead>
<tbody>
<tr>
<td><strong>What is it? (definition)</strong></td>
</tr>
<tr>
<td>• Per hundred</td>
</tr>
<tr>
<td>• Fraction with denominator 100</td>
</tr>
<tr>
<td><strong>Properties/Facts:</strong></td>
</tr>
<tr>
<td>Can be expressed as a fraction, decimal, ratio</td>
</tr>
<tr>
<td><strong>Comparisons:</strong></td>
</tr>
<tr>
<td>• Fractions</td>
</tr>
<tr>
<td>• Ratios</td>
</tr>
<tr>
<td><strong>Examples: (found in real life)</strong></td>
</tr>
<tr>
<td>• Discounts</td>
</tr>
<tr>
<td>• Commission</td>
</tr>
<tr>
<td>• Interest rates</td>
</tr>
<tr>
<td>• Test scores</td>
</tr>
<tr>
<td>• Statistics</td>
</tr>
</tbody>
</table>
**Semantic mapping** or thought web: Students break down a concept into related sub-topics and list examples or characteristics under each sub-topic. (See 'Sharing Recipes', pg. 69.)

*Frayer model* (Frayer, Frederick & Klausmeier, 1969): The Frayer model uses four categories to help students understand a concept. Similar to concept mapping, students write a definition in their own words, list characteristics, list examples and non-examples. (See 'Sharing Recipes', pg. 70.)

<table>
<thead>
<tr>
<th>Definition:</th>
<th>Facts:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Any number that has three or more factors.</td>
<td>Numbers with an odd numbers of factors are perfect squares.</td>
</tr>
<tr>
<td>25: The factors of 25 are 1, 5, 25.</td>
<td>25: The factors of 25 are 1, 5, 25.</td>
</tr>
<tr>
<td>36: The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 16, 32.</td>
<td>36: The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 16, 32.</td>
</tr>
</tbody>
</table>

**Composite Numbers**

<table>
<thead>
<tr>
<th>Examples:</th>
<th>Non-examples:</th>
</tr>
</thead>
<tbody>
<tr>
<td>4: factors of 4 are 1, 2, 4.</td>
<td>Prime numbers – a number that only has two factors, one and itself.</td>
</tr>
<tr>
<td>24: factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.</td>
<td>2, 3, 5, 7, 11, 13, 17, 19.</td>
</tr>
<tr>
<td>35: factors of 35 are 1, 5, 7, 35.</td>
<td>0 and 1 are not composite numbers.</td>
</tr>
<tr>
<td>48: factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, 48.</td>
<td></td>
</tr>
</tbody>
</table>
Concept Rating Chart: To survey prior knowledge, the teacher can list a series of terms and ask students to self-assess their understanding using a simple checklist similar to the one below.

<table>
<thead>
<tr>
<th>Concept Rating Chart: Rate the following terms using the statements below.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I've never encountered this term before.</td>
</tr>
<tr>
<td>2. I've read this term before but I don't understand what it means.</td>
</tr>
<tr>
<td>3. I've read this term before and I think I could apply it.</td>
</tr>
<tr>
<td>4. I understand the meaning of this term and I am confident that I can apply it to solve a problem.</td>
</tr>
</tbody>
</table>

Word sorts: Writing vocabulary on separate cards and encouraging students to sort vocabulary into categories engages students to classify words in a way that is meaningful to them. Helping them to make connections through attributes will deepen their understanding of the concepts.

Number Sorts: Using a Venn diagram template can help students classify and sort attributes of numbers.

![Factors of 24 and Factors of 45 Venn Diagram]

Factors of 24

Factors of 45

Common Factors

*K-N-W-S Chart*: This graphic organizer allows students, after reading a problem, to list: what facts they Know about the problem, what information they do Not need, What the problem asks them to find, and to describe what Strategy or strategies they will try.

(See ‘Sharing Recipes’, pg. 71 for the K-N-W-S template.)

*Five Step Process Chart*: This graphic organizer allows a struggling reader to gather information related to a specific problem. It is very prescriptive
and may inhibit students from looking at various strategies. It is a good technique for special needs students who may need a scaffolded approach. (See 'Sharing Recipes', pg. 72 for the Five Step Process Chart template.)
Creating and Following Recipes

The Problem Solving Process

The challenge with ‘teaching’ problem solving is that by definition, if you are faced with a problem, you don’t know after the first reading how it can be solved. Problem solving can’t be taught - it has to be practiced! By encountering and solving many types of problems, students gain experience and consequently, will develop a ‘glossary’ of strategies to try with each subsequent problem. Through success, and failure, the strategies will be modified and defined as a process.

When introducing a problems-based approach in the classroom, model the process of problem solving. Use a familiar problem to demonstrate the three stages of the process: the ‘wonder’ stage, the ‘working’ stage and finally, the ‘review’ stage. Model the questioning strategies for each stage. Consider posting these questions on a bulletin board as a reference for students.

The ‘Wonder’ Stage:
During the first encounter with a problem, a student can be said to be in a ‘wondering stage’. Not only does the student have to read and decode the words and symbols, but it is also a time to ask questions about meaning and the conditions of the problem.

Questions to ask in the ‘wonder’ stage:

• What do I know from reading the problem?
• Do I need any other information? How will this help me understand the problem better?

• Am I making any assumptions about the problem? Is there another way to look at the conditions of the problem?

• How can I organize the information I already know? Can I list the information, draw a picture, create a chart or a table, graph the information?

• What am I asked to find? Can I restate the problem in my own words?

• Can I guess at the answer or estimate an answer? Can I simplify the problem or work on part of the problem?

• I wonder if 'this' strategy will work?

(Adapted from Menu Collection: Problems from Mathematics Teaching in the Middle Grades. Patrick Collier - editor)

The 'Working' Stage:
The working or 'attack' stage of the process consists of more than deciding on a plan and carrying out the plan. It is a cycle of attempting to find a solution and, if the plan didn't work, going back to the 'wonder' stage to gain a better insight into the problem. It can take many attempts to solve the problem!

Questions to ask in the 'working' stage:

• If I make a guess or conjecture, can I check to see if it is true? If the guess is true, how can I use this to solve the problem? If the guess is false should I abandon the conjecture or try to modify it and test again?
• Can I list some examples that fit the criteria and then look for a pattern?
• If I get frustrated, have I looked back at the problem to review the information? Can I try a different way to organize the information? Can I restate the problem in a different way to help me look at the problem from another perspective?
• Have I considered other possible strategies, such as, eliminating possibilities, working backwards, solving a simpler problem, using an algebraic expression, acting it out, conducting an experiment, using manipulatives, using technology, sketching a diagram or making a model?
• What have I tried that hasn't worked? Why didn't it work? Did I make a calculation error? If I tried it again would I get the same results?

(Adapted from Menu Collection: Problems from Mathematics Teaching in the Middle Grades. Patrick Collier – editor)

The 'Review' Stage:

Once a possible solution has been determined, the third stage of the process is to 'review' the solution. Students should be encouraged to look at this stage as a chance to justify their solution or to find a better way to solve the problem. Often students feel this stage is as simple as asking, “Is the answer right?” Reflecting back and thinking about the process may be as important, if not more important, then actually determining a solution!

Give students time to reflect back with a partner or as a class, as well as individually. Dealing with the frustrations or obstacles, coupled with
seeing a variety of possible solutions will help students gain insight and consolidate connections!

Questions to ask in the ‘review’ stage:

- Do I have a solution? Can I justify and explain my answer to the teacher or a friend?
- Did I communicate my solution effectively in writing? Did I use words, numbers and pictures to help me communicate?
- Can I find a different way to solve the problem? Is there an easier way?
- What did I learn about math through this challenge? What did I learn about problem solving? What did I learn about myself?
- What do I still wonder about?
- Can I ask an interesting ‘what if’ question? Can I make up my own problem related to this challenge?

(Adapted from Menu Collection: Problems from Mathematics Teaching in the Middle Grades. Patrick Collier - editor)
No Nibbling!

The Hardest Part of Teaching

The hardest part of teaching may be *not* giving students the answers! You may be the best at explaining mathematical concepts but if your students can’t explain the concept in their own words and can’t apply the skills, then learning hasn’t taken place! There is a shift needed so that students, not the teacher, can explain a concept so well that they demonstrate their learning.

In moving towards a more problem-centered classroom, there are a few changes you, as the facilitator, can implement to send a clear message to your students that the thinking process is essential and their active participation is a vital component for learning to take place. Some of these suggestions are not easy to consider since, as educators, we are so excited about giving information and trying to ‘show’ students how to solve a problem. We need to step back and realize that we can’t ‘turn on the light bulb’ for them! Students need to make the connections - we can guide and make sure we ask the right questions.

- Consider not giving direct instruction to students! Ask good questions instead!
- Ask open-ended questions if possible, where more than one solution is possible.
- Explain that responses to questioning will lead to insight into understanding and determines the next steps in the process.
• Be patient when asking a question. It sometimes seems easier to give the students the answer but in the long run they will learn more by struggling to obtain a solution and being able to justify their responses.

• Being an active listener during discussions can help you pinpoint students who have good ideas. Call on them to give the response when the class reconvenes as a group.

• Ask students to paraphrase and analyze each other’s solutions. If you, as the teacher, accept the first response and show the errors or inconsistencies, then you’re sending the message that you will do the thinking for the students. Strive to have students be the ‘detective’ looking for clues to understanding.

• If a student asks, “Is this the right answer?” consider asking the student to defend or justify his/her answer rather than saying yes or no. (You may even want to pretend that you don’t know the answer!) If there is an error, the student may find the answer for himself/herself or you can ask a question that makes the student rethink the solution. If the student does explain the correct solution, ask a ‘what if’ question so thinking doesn’t stop.
Add a Dash or Pinch

Asking Questions

Effective questioning helps to promote math dialogue and helps students clarify thinking, consider different solutions, predict what might happen if a condition changes and justify answers. “Teacher responses and questions that encourage a child to pursue inquiry, skill development and problem solving are those that help the child revisit and extend a learning experience.” (S.L. Schwartz. *Hidden Messages in Teacher Talk: Praise and Encouragement*) Listed below is a series of questions. Consider posting these questions on a bulletin board for students to review periodically or to be used as starters for learning log entries.

Asking Students to Reason or Conjecture:

- Is that true in all cases?
- Can you think of another example?
- Can you think of a situation where this solution would not work?
- What would happen if….?*
- Do you see a pattern?
- Can you predict what the next term in the sequence will be?
- What are the similarities and differences of the two solutions?
- What assumptions are being made in solving the problem?

Asking Students to Think about Connections:

- How does this relate to…?
- Have you ever solved a similar problem before?
• What concepts have we learned in previous activities that could be used to solve this new problem?
• Can you give me another example of...?
• How does this concept relate to the everyday life?

Asking Students to Justify:
• Why do you think that?
• How did you reach that conclusion?
• Can you explain this in another way? Can you model this in a drawing?
• Can you convince your group that your solution makes sense?
• Does anyone have the same answer but has explained it a different way?
• Would the same thing happen if you tried it again?

*Asking ‘what if’ questions helps to extend learning by continuing the process of investigation. Finding one solution will not be the stopping point if another question is posed that springboards off the initial information! Encourage students to ask these types of questions as they solve problems. Here are some examples of ‘what if’ questions:
• What if you double the length and width the sides of a rectangle? How will the area be affected? How will the perimeter of the two rectangles be related?
• What if you multiply the diameter of a circle by three? How is the area affected?
• What if 4 is added to each of five numbers? What happens to the mean?
• What if you apply the GST to the total, then apply the PST to the total? Is this the same as adding the GST and the PST together and applying this percent to the total?

• What if you doubled the length of each edge of a cube? How would the volume be affected?

• What if you turned mixed numbers into improper fractions, then added? Would you get the same result as adding the whole numbers of the mixed fractions, then adding the proper fractions together?
Appetizers:

Warm-up Activities

The best way to encourage students to attempt complicated, rich tasks, is by ensuring success and building confidence through simpler, quick activities. Students will practice using strategies and thinking critically to solve these problems. Having a 'Problem of the Day' posted as students walk into class promotes a routine of getting ready for class and using every possible minute. Warm-up activities can relate to the concept being studied in class that day or can connect to other strands.

You may want to set a time limit for the activity before solutions are discussed in pairs or as a class. (You may decide not to discuss solutions and assign the problem as homework for further investigation!) Have a second problem ready for students who finish early or set up learning centers for students to investigate problems. Use strong problem solvers to act as peer helpers using paired-reading or reciprocal teaching strategies.

Students can use journals or learning logs to explain their reasoning. Remind students that being 'stumped' doesn't mean an opportunity not to think! Encourage students to write down questions to help them pinpoint what they don't understand about the problem.

(Solutions to 'Warm-up Activities' can be found in 'Secret Ingredients', pg. 87.)
Sample Questions:

* (Many of the Appetizer sample questions are adapted from What You Do When You Don’t Know What To Do. Grant Jones)

Tiling: Tiling activities can be made for every strand in math. Using the digits 1 through 9, students place the tiles appropriately to make the equations true. It is a wonderful risk-taking activity since the tiles can be moved easily without the worry of errors and starting over again. The activity reinforces the need to reflect back on their work and justify their solutions. Here are two examples. The template for this activity is found in ‘Sharing Recipes’, pg. 73 and 74.

1. Tiling

   Greatest common factor of 18 and 45
   a multiple of 10
   a prime number
   the prime factors of 30
   \[ \times \times \times \]

2. Tiling

   \[ \begin{array}{c}
   7 \\
   \times \\
   4 \\
   \end{array} \]

   \[ \begin{array}{c}
   2 \\
   3 \\
   6 \\
   \end{array} \]

   \[ \begin{array}{c}
   + \\
   3 \\
   3 \\
   6 \\
   \end{array} \]

3. Place one of the numbers 1, 2, 3, 4, 5, and 6 in each circle. The sum of the 3 circles on each side of the triangle must total 10.

   What if the total had to be 12?
4. Selena threw 3 darts. If her total was 29, what possible combinations were thrown?

5. During the 5 days of March Break, Brad looked after the neighbour’s children. Each day he earned $2 more than the previous day. He earned a total of $60. How much did he earn each day?

6. Change the letters to numbers. Each letter that is the same must be the same number.

   A A A   Is there more than one solution?
   A A A
   A A A
   + A A A
   B  A  A  C

7. Siying is now 3 times as old as her daughter Siu. In 11 years Siying will be twice as old. How old are the mother and daughter now?
8. Place the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 in the squares so that each row, column and the diagonals total 15. Look at your answer compared to someone else. What do you notice?

![Sudoku puzzle]

9. A furniture store sold tables and three-legged stools. There were 68 legs altogether. How many of each table or stool did they have in stock? Is there more than one solution?

10. The area of a rectangular room is 18 m$^2$. The perimeter of the same rectangle is 18 m. What is the length and width of the rectangle? Is there another shape that has the same number of units in the area as the perimeter?

11. Join all 9 dots using only four straight lines segments without lifting your pencil from the paper.

```
 .   .   .  
 .   .   .  
 .   .   .  
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12. Remove 1/4 of the toothpicks and replace the toothpicks to leave only 3 completed squares.

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13. Arrange 14 chairs in a square room so that there will be the same number of chairs along each wall.

14. You have just received nine rare coins. You get an anonymous tip that one of them is counterfeit. The real coins weigh 10 grams each. The counterfeit looks like the genuine coins, but it weighs less. The only scale you have is an old balance scale with two pans, one on each side of a balance point. What is the fewest number of weighings you can make and identify the counterfeit? Would your answer change if you knew that the counterfeit coin was heavier than the genuine coin?

(Mathematics: Teaching in the Middle School. Oct. 1994 No. 6)

15. In a 100 m race, Judy finished 1 m ahead of Mark.
   Jalal finished ahead of Judy, but behind Jennifer.
   Melanie finished 4 m ahead of Mark and 1 m behind Jalal.

   What was the finish place of each runner?

16. Eight students played in a round robin badminton tournament where each student played every other student once. How many games were played?

17. Imagine cutting a pizza with 4 straight lines. The cuts do not have to go through the center - but they could. The pieces do not have to be the same size - but they could be. Show the fewest pieces possible with four cuts. Show the most pieces possible with four cuts.

18. Sherry went to a store and bought a pencil for 23 cents. She paid the exact amount and received no change. How many different ways could Sherry have paid for the pencil?

(Adapted from Probability Activities. Robert Lovell.)

19. Lance bought some doughnuts and gave \(\frac{1}{2}\) to Merry Lee. Merry Lee gave Ben half of what she got from Lance. Ben gave \(\frac{1}{2}\) of what he got to Kelly. Kelly received 3 doughnuts. How many doughnuts did Lance buy?

20. How many numbers less than 50 use only odd digits?
Smorgasbord!

Samples of Problem Solving Activities

The activities presented in this section area are a mere starting point for a collection of challenging problems. Start a network of sharing problems with colleagues. Ask students and parents to contribute to the problem bank!!! (Solutions to 'Warm-up Activities' can be found in 'Secret Ingredients', pg. 90.)

Marshmallow – Toothpick Challenge
Use 21 toothpicks and 14 marshmallows to build a structure that will support the weight of a videocassette.

Palindromic Reversals:
The year 2002 is a palindromic number (A number which is the same if written from left to right or right to left!) A palindromic number can be created when a number is reversed and added to the original number. It may take several reversals, but the process will lead to a palindromic number! Look at the following examples:

Start with the number 13. Reverse the number and add. The result is a palindromic number obtained with only one reversal. 13 + 31 44

Try the number 19. 19 First reversal 44 + 91 110

The first reversal does not produce a palindrome so another reversal must be executed using the sum from the first reversal.

110 The second reversal + 011 produces a palindrome. 121

Challenge: How many reversals will it take to create a palindromic number, starting with the number 89?
Calendar Challenge:
A. Draw a square around any four numbers on a calendar. Can you recognize and describe a number pattern with these numbers? Will the pattern apply to other boxes of four numbers on the calendar?

<table>
<thead>
<tr>
<th>SUN</th>
<th>MON</th>
<th>TUES</th>
<th>WED</th>
<th>THURS</th>
<th>FRI</th>
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<td>30</td>
<td>31</td>
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</tbody>
</table>

B. Draw a square around any nine numbers on the calendar. Find the sum of the nine numbers and divide by nine. What did you notice? Repeat this with other blocks of nine numbers. What pattern do you see? Explain why it is always true.

(Adapted from OAME ABACUS. Dec. 1999)

Jelly Bean Jars:
Maria, Jason and Kyle each tried to guess the weight of a giant jar of jelly beans. Maria’s estimate was 4.7 kg. Jason’s estimate was 8.2 kg and Kyle’s estimate was 11.4 kg. One estimate was off by 1.3 kg, another by 2.2 kg and another by 4.5 kg. How much did the jelly bean jar weigh?

Partition of a Square:
A square measuring 10 centimeters on a side is divided by vertical and horizontal line segments into rectangles with areas of 12, 18, 28, and 48 square centimeters. Where should the vertical and horizontal lines be located?

(Mathematics: Teaching in the Middle School. Sept. 1998 No. 13)
Painting Buddies:
Matthew and Allison were paid $60 to paint the garage. Matthew started at 8:00 am, and Allison did not arrive until 10:00 am. The work was completed at 2:00 pm. What is Allison’s fair share of the earnings? (Mathematics: Teaching in the Middle School. May. 1998 No. 5)

Equivalent Fractions:
The fractions 3/6, 7/14, and 29/58 are equivalent, since each is another name for ½. As a set, these fractions use all the digits from 1 through 9 once. Find another set of fractions similar to this set, using 3/6, again, as the first fraction of the set. (Mathematics: Teaching in the Middle School. Mar. 1996 No. 14)

Investigate Powers:
Which is greater 2\(^1\) or 1\(^2\)? 3\(^2\) or 2\(^3\)? 4\(^3\) or 3\(^4\)? 7\(^6\) or 6\(^7\)? Investigate the pattern so that you can predict a general pattern. (Mathematics: Teaching in the Middle School. Dec. 1994 No. 7)

Movie Promotion:
At a recent movie presentation, a free hat was given to every eighth student and a T-shirt was given to every twelfth student. Who were the first three students to each receive both a hat and a T-shirt?

Pass the Cookies:
Jennifer, Ahmil and Louise had eight cookies to distribute among themselves. They did not need to get the same number of cookies, but they must all get at least one cookie. In how many ways could the cookies be distributed? (Mathematics: Teaching in the Middle School. Dec. 1997 No. 13)

Number a Cube:
Number the eight corners of a cube from 1 to 8 so that the sum of the four corners of each face is 18. (Mathematics: Teaching in the Middle School. Jan. 1997 No. 5)

Tip the Driver:
A group of friends took a bus trip. Each traveler gave the bus driver a tip using the same nine coins. The total tip was $8.41. How many dimes did the driver receive? (Mathematics: Teaching in the Middle School. Apr. 1996 No. 16)
Pizza Pizza:
A waiter in a pizza restaurant said that the medium pizza was 25 centimeters in diameter and the large pizza was 40 centimeters in diameter. He also said that the 40 centimeter pizza was about twice as large as the 25 centimeter pizza. Was he correct?  
(Adapted from Mathematics: Teaching in the Middle School. Dec. 1994 No. 16)

Red, Green and Blue:
A bag contains five red marbles, three green marbles, and four blue marbles. How many green marbles must be added so that the probability of drawing a green marble is \( \frac{1}{3} \)?  
(Adapted from Probability Activities. Robert Lovell.)
Rich Learning Tasks

An activity that fully engages students in a task creates a climate of excitement and anticipation in a classroom. In this type of authentic activity, every student regardless of his/her ability level can tackle the problem to some extent, because it has meaning in his/her life. There is an opportunity to investigate, explore, discover and relate to previous experiences. It is the kind of activity that nurtures independence or working as an interdependent team member. Habits necessary for life-long learning and 'sense-making' skills are fostered. We are confronted by an affirmation and realization that students learn by actively being involved. (Notes taken at OMCA meeting, Oct. 2000; Gary Flewelling - guest speaker)

"Young children are active individuals who construct, modify, and integrate ideas by interacting with the physical world, materials and other children. Given these facts, it's clear that ...learning... must be an active process." *NCTM, 1989*

Rich learning tasks have many of the following characteristics:

- **Curriculum Relevance** (linked to expectations);
- **Student relevance/Student engagement** (meaningful to students);
- **Flexible Structure** (the structure of the problem does not impede creativity);
- **Context/Authenticity** (a real-life role and situation);
- **Flexible entry level** (every student should be able to begin the problem; some students will need accommodations, such as, scaffolded questioning);
- **Performance rubric** (students should know the assessment criteria);
- **Opportunity to problem solve/inquire/investigate/experiment/explore**;
- **Opportunity to model/formulate/transform/manipulate**;
- **Opportunity to use imagination**.
• Opportunity to organize information/communicate;
• Opportunity to interact;
• Opportunity to infer/argue/prove/conclude;
• Opportunity to pose problems;
• Opportunity for extensions and enrichment;
• Opportunity to reflect on learning/self-assess.

Adapted from “A Handbook of Rich Learning Tasks” Gary Flewelling

A worthwhile collaborative project for teachers at a school to undertake would be to choose a traditional task or activity and, by using the above guidelines, attempt to transform the activity into a richer task. Often by looking for the enduring understanding you want the students to glean from the activity, then rewording the situation or question, the activity takes on a greater probability that learning will take place. Critically looking at the physical structure of the task (existing worksheets that may impose a strategy) and giving students more freedom to organize their own investigations will enhance the task even more.

Here is an example of a traditional task, applying the rules of order of operations, which was transformed into a richer task. Rather than using a textbook exercise or a blackline master, students worked in pairs with a deck of standard playing cards.

**Order of Operation Challenge**

**Exploring the task:**
Deal out two cards. Using these two cards and any operation sign, including +, -, x, ÷, !, brackets, square roots, plus using a card as a base or the power of an exponent, construct as many equations as possible.
Exploring the task:
- Organize your work to show the possibilities.
- Reflect on your calculations.
- Reflect on the math vocabulary you discussed while investigating.
- Did you have any 'What if' or 'I wonder' questions?

Investigating the task:
Keep the same two cards and add a third card. Using these three cards and any operation sign construct as many equations as possible.
- Organize your calculations.
- List your observations.
- Report any new math vocabulary introduced as you were investigating.
- Did you have any 'What if' or 'I wonder' questions?

Extending the task:
Keep the same three cards and deal two more, making a total of five. Then deal one last card to act as the target card. Using all five cards and any operation sign, including +, -, x, ÷, !, brackets, square roots, plus using a card as a base or the power of an exponent, construct an equation that will result in the value of the target card.
- Record your observations and calculations.
- Is there more than one possible solution?
- What if you kept the same six cards and used each card as the target card for each subsequent attempt?

Presented by Lin Dicksen at the IDEAS Conference 2000

Although structure was imposed during this activity, it was not as rigid as a worksheet. Every student would be engaged in at least the first stage of the problem. Even in the first stage, there would be the possibility of asking what if questions and demonstrating connections. Depending on the deal of the cards, students may have discussed fractions, integers, decimals or exponents. They may have developed a pattern to determine the greatest
number of combinations. The way they organized the investigations was left up to them.

Adding a third card, would have increased the difficulty of the activity. There would have been an increase in the number of calculations and the 'math talk', AND the activity was more engaging than working on a blackline master of thirty questions!!

**Other Examples of Rich Learning Tasks:**

**Math in Children's Literature:**

**Assignment:** You have been hired by a publishing company to write a children's picture book that helps to teach math concepts. Research a variety of books to determine the characteristics of a good book. Follow the writing process to write your story. Consider how you will illustrate your book. Publish a good copy then celebrate by visiting a class of younger students and reading your book to them.

**Bedroom Box Project:** You are an architect who has been commissioned by a family to create a scaled model of their teenager's bedroom. Research the costs of decorating the room, including window coverings, tiles or carpets, paint or wallpaper. Include your plans and calculations.

**Earning a 'Little' Extra Money:**

You need to earn a little extra money to help buy a new bike. You offer to do household chores for a retirement home every day for 30 days. You are offered a set rate of $100 for the month or you could accept another payment plan. You would work for 1 cent the first day and double that
amount each day for 30 days. Which payment plan would be to your advantage? Justify your reasoning.

**Quilting Patterns:**
A. You are asked to research traditional quilting patterns for a local historical society. Analyze the patterns by looking for geometric relationships used in colour variations. Present a report to the historical society to explain the mathematical significance of certain traditional patterns.

B. The historical society is promoting the connection between the past and the present by promoting a contest for a modern day quilting pattern, symbolic of Canada today. Create a pattern and explain the significance of your design.

**Math Rally:**
You are a teacher who would like to assess the problem solving skills of his/her students in a particular strand of math by creating a game called 'Math Rally'. This game will be similar to a car rally. Students will be given a map and a starting point on the map. You create 15 multiple choice questions, with one correct answer and two false answers for each question. For each choice, you provide a set of directions, for example, 'If the answer to the question is 'a' then travel south past two intersections and wait for further instructions. If the answer is 'b' then turn left at the next intersection and stop.'

Create the game, making sure you have the correct path! Exchange the game with another group of students and see if they get lost!
**Graduation Dance:**

You are on the student council committee responsible for planning the Grade 8 graduation dance. The parent council has donated an amount of $600 for the evening. Plan and budget for the event. Prepare a report for the parent council detailing how the money will be spent.

**Humanitarian Aid Project:**

Your school has decided to raise money to send to students in a third world country devastated by a hurricane. Your class has set a goal of raising $200 for your contribution. You are the chairperson of the committee. Your class has already decided to make batches of Chocolate Crispy Squares.* Your job is to decide how much to sell each piece and to calculate how many batches to make in order to meet the $200 quota and pay the costs of buying the supplies.

<table>
<thead>
<tr>
<th>*Recipe for Chocolate Crispy Squares</th>
<th>Cost of Supplies</th>
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</thead>
<tbody>
<tr>
<td>2 cups chocolate chips</td>
<td>4 cups of chocolate chips cost $3.50</td>
</tr>
<tr>
<td>½ cup margarine</td>
<td>4 cups of margarine cost $4.00</td>
</tr>
<tr>
<td>½ cup corn syrup</td>
<td>6 cups of corn syrup cost $2.00</td>
</tr>
<tr>
<td>1 cup icing sugar</td>
<td>4 cups of icing sugar cost $2.00</td>
</tr>
<tr>
<td>6 cups cereal</td>
<td>24 cups of cereal cost $5.00</td>
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<tr>
<td>*Each recipe makes 32 squares.</td>
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</table>
**Mini-Mall Project:**

You are hired by a craft store to develop and market a new product. Working with a design team of three other people you brainstorm a list of possible products that would interest the demographics of an intermediate school. After surveying the market, you decide on a product, have it approved by the manager and are given a start-up budget. You must produce a minimum quantity of your product to test market. Before the product can be tested, an advertising brochure is produced to accompany the display at next month’s trade show.

**Driveway Tessellations:**

You are an interlock contractor who has been hired to redo a driveway and walkway. The customers have requested that the design be of two different colours and sizes. Create a pattern and a scale drawing as a contract proposal. Include the cost of the brick and labour charges in your proposal.
Writing in Math

Writing in math is not easy for many grade 7 & 8 students. As students work through the process and respond to guided questioning, they will become more comfortable. Since there is an emphasis on presenting solutions in a variety of ways, the use of diagrams, models and graphic organizers will help many students who have difficulty expressing their solutions using written language.

Model the process. Explain the strategies you use to decode information and pick a strategy. Teach and model how to use numbers, words and pictures to show how you solved the problem. Teach and model how you can 'show another method'. Model how you reflect back and justify your reasoning.

Early in the year, or as needed, have students practice writing a solution using a familiar context. Have students work in pairs initially to help clarify questions and the process as they work together. Follow-up with individual practice using similar problems before introducing complicated multi-steps problems.

The use of learning logs and journals are invaluable tools to demonstrate understanding by the student and for the teacher to gain insight into the student’s learning. Providing timely feedback to student entries is crucial since this opens a dialogue. Many intermediate students are masters at hiding their problems. They may reveal their frustrations and
what they don't understand in writing. Assure students that journals are private documents and will be respected.

Learning logs or journals can be used to map out and explain solutions or respond to challenges:

- Write a poem that demonstrates your understanding of numerators and denominators;
- Assume the role of a teacher who must prepare a lesson on experimental probability;
- Write a multiple choice question and explain why a student might choose the wrong answers you've listed;
- Write a letter to your pen pal explaining what you discovered about the sum of the interior angles of triangles;
- Write a letter to your friend who has confided in you that he/she doesn't want to take math in high school;
- Design a poster to give examples of how to add fractions;
- Write a song to explain the order of operations.

Learning logs or journals can be used to comment on mathematical issues, attitudes, concerns or reflect back on learning:

- My favourite part of math is...;
- I have difficulties when asked to...;
- I was confused about...;
- Describe how you use math in your everyday life;
- Explain what strategies you use to read a difficult problem. How do you get started?
- Describe a fascinating pattern that you discovered;
- Describe one skill you would like to improve in math class;
• Two things I would like you to notice about my solution are...
• One thing I know that the group did not mention;
• I used to think ...., but now...
• I’d like to know more about ...;
• When I hear someone say Math is fun, I ...;
• Math is like....
• Reflect back on this activity. Complete the following phrase:
  o The most difficult part was ..., 
  o The easiest part was ..., 
  o I learned that I..., 
  o I was surprised that I..., 
  o I discovered that I..., 
  o I was pleased that I ...
When the focus in Mathematics was geared to basic skills and mastery learning, assessment of paper and pencil tests, quizzes and assignments was perceived as easy - the answer was either right or wrong! But what did these assessment strategies really assess? Is it possible that it only gave us a limited glimpse - a one-dimensional view - of understanding? Rote learning and memorized procedures of what to do would soon be forgotten.

In an effort to give students the chance to make sense of math, rich tasks that are open-ended and have more than one solution place a responsibility on the students to communicate their responses. Assessment may now be perceived as more difficult but the information gained paints a multi-dimensional and colourful portrait of the learner's understanding!

Consider a shift in thinking about the learner and the levels of achievement. In his book, *A Handbook of Rich Learning Tasks*, Gary Flewelling suggests that a student achieving in level one should be considered as a **novice** at understanding and applying a concept. After gaining experience the student becomes an **apprentice** and is starting to approach the expected standards. A student who has achieved the provincial standards is said to be a **craftsperson**. A student who shows insight and has made sophisticated connections can be compared to a **master craftsperson**. This is a less negative way to look at the levels and suggests a process of development.
The most important tools in a problems-based classroom are observation, active listening and anecdotal records. To be an 'active listener' suggests that educators:

- Watch carefully as students investigate and explore;
- Listen to how they explain concepts to their peers and to you;
- Listen to the questions they pose;
- Observe what they draw and how they represent concepts visually;
- See how they react when a problem is given;
- Read their learning logs and explanations carefully for insight.

Create a system to record observations and make quick notes as needed. Use stick-on labels to write observations then ‘stick’ the labels on a tracking sheet. Use recipe cards to record information. Palm pilots are recent tools that can be used to record observations, then downloaded into the computer. Observation charts, linked either to the learning skills categories or specifically to problem solving can be used to record observations on a regular basis. (See ‘Sharing Recipes’, pg. 75 and 76.)

As students are working, dialogue with them and record general impressions. Schedule a more formal conference with students to discuss a concern or as a verbal demonstration of understanding. This could replace a traditional paper and pencil test! (See an example of ‘Conference Recording’ sheet in ‘Sharing Recipes’, pg. 77.)

Explain the importance of self-assessment and give students the chance to assess their perceptions during problem solving challenges. (See the template of ‘Thinking About My Problem Solving’ in ‘Sharing Recipes’, pg.
78.) Journal writing and learning logs, as discussed in an earlier section, are extremely important sources to gather samples of learning.

At the beginning of the year review the elements of problem solving with students. Refer to the general rubric found in 'Sharing Recipes', pg. 79. This rubric would also be appropriate to help parents understand assessment. Include this type of rubric when sending a math portfolio home or during student-led conferencing.

Before each problem-solving task, review the criteria for assessment. Allow students the chance to give suggestions on how they will be assessed. This allows students to take ownership of their work and to better understand the expectations.

Give verbal feedback to students on an on-going basis. Written comments, which highlight what the student did well, what improvements could be made and considerations for 'next steps', will help the student focus on the cycle of learning. This feedback can be incorporated to assist the student improve on a future activity.
Everyone has to work together in a problem-based classroom. If you value learning as an active process then participation is not optional! Whether you are working as a whole group, in small groups or in pairs expect that every student will contribute. Use instructional strategies that will encourage participation.

- Use a ‘Think-Pair-Share’ strategy regularly to get everyone in the class to participate. After posing a problem, allow students time to work on their own, then ask students to pair up and discuss their solutions. Two pairs can then join together or solutions can be presented to the class.

- Use hand signals to encourage participation and help you survey the class. When brainstorming solutions, giving a ‘thumbs up’ to indicate agreement, a ‘thumbs down’ to indicate disagreement, or a ‘horizontal’ hand position to indicate that the student is not sure, is a great way to ensure every student is on task and participating.

- Explain to students that not having a solution is quite possible but to stop thinking and allow someone else to give you the answer is not acceptable. You see this kind of thinking in the response, “I don’t get it!” Expect that students ask questions which help to pinpoint where they are having problems.
• Encourage students to have more than one answer ready. While soliciting answers, get into the habit of asking, “Any other possibilities? What did you think about that suggestion? What if…”

• Encourage group responsibility by expecting that all members of the group be able to explain the solution. Call on different members of the group.

• Encourage students to contribute their fair share of the workload. Groups that understand the strengths and weaknesses of individuals, especially special needs students, may recognize the subtle implications of ‘fair share’ compared to ‘equal’ workload. Use a ‘Cooperative Work Contract’ at the beginning of an activity to help organize. At the end of each work period, students can reflect back on their successes, frustrations and plan alternative strategies. (A template for contract is found in ‘Sharing Recipes’, pg. 80.)

To encourage maximum participation, structure activities to promote group cooperation. Here are examples of activities that focus on cooperative learning.

**Math Olympics:**

Groups of three or four students form an alliance to represent a country and compete in Math Olympics! Allow for team building activities, such as, creating a flag and motto for each country. Problem solving challenges can be given to the teams and points awarded for correct solutions. It is advisable to modify the concept of awarding points to the team that finishes first, second and third by imposing a time limit and giving
points for correct solutions, regardless of how quickly the team performed. Questions that have more than one solution are preferable since groups can continue to look for possible solutions until the time elapses.

   Careful thought must be given to selecting groups to avoid 'international incidents' and ensuring fair play among nations! Keep the nations together for a few months before having to redraw the boundaries!

   **Cooperative Sketching:**

   As a vocabulary building activity, create several simple geometric designs based on concepts reviewed in class, (see example below), then cover the design with a piece of card stock. Working in pairs and sitting back-to-back, one student describes the drawing while his/her partner tries to draw the design - without peaking, of course! After completing the design, partners look at the drawing and discuss how they could have improved. Students reverse the roles, using a different design.

   ![Sketching Examples](image)

   **Math Detectives:**

   In groups of four, students work together to eliminate possibilities of number suspects to find the culprit! Each student has a unique clue on his/her worksheet and must contribute this information so the group will be
successful. This kind of activity ensures that all students in a group participate equally.

Pictured below is the first case to be solved. Refer to 'Sharing Recipes' for the template, pg. 81-84. This activity could be used early in the year to introduce cooperative learning and to highlight one of the themes!

**Student #1:**

**Case 1:** Number suspects: 3, 4, 5, 6, 7, 8, 9, 10  
**Clue # 1:** The number is an even number.

**Student #2:**

**Case 1:** Number suspects: 3, 4, 5, 6, 7, 8, 9, 10  
**Clue # 2:** The number is not 4 x 2.

**Student #3:**

**Case 1:** Number suspects: 3, 4, 5, 6, 7, 8, 9, 10  
**Clue # 3:** The number is not 12 ÷ 3.

**Student #4:**

**Case 1:** Number suspects: 3, 4, 5, 6, 7, 8, 9, 10  
**Clue # 4:** The number is a greater than 8.

By the process of elimination and working together the group determines that the culprit is number 10!

**Go to Your Corners:**

In the Math detective activity, students must work together to give each other clues but there is always the possibility that one student will
monopolize two worksheets or one student will be reluctant to read his/her clue. The activity 'Go to Your Corners' goes one step further in ensuring equal participation. (This activity should be introduced later in the year.) In each corner of the room a clue or question is posted. Each student in the group is assigned a clue and must go to the corner, read the message and bring back the information to the group. They may not write down the message as they read but as soon as they return to their seats they may jot down the information. (Special needs students may need to have a copy of the clue to take back to the group.) Students work together to solve the questions posed.

One probability challenge is diagramed below. Refer to 'Sharing Recipes', pg. 85 for other 'Go to Your Corners' challenges based on probability.

<table>
<thead>
<tr>
<th>“Go to Your Corners”</th>
<th>“Go to Your Corners”</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problem Set 1</strong></td>
<td><strong>Problem Set 1</strong></td>
</tr>
<tr>
<td><strong>Clue 1</strong></td>
<td><strong>Clue 1</strong></td>
</tr>
<tr>
<td>M A T H E M A T I C S</td>
<td></td>
</tr>
</tbody>
</table>

**Situation:** A card is drawn at random

<table>
<thead>
<tr>
<th>“Go to Your Corners”</th>
<th>“Go to Your Corners”</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problem Set 1</strong></td>
<td><strong>Problem Set 1</strong></td>
</tr>
<tr>
<td><strong>Question 1</strong></td>
<td><strong>Question 2</strong></td>
</tr>
<tr>
<td>P(~E)</td>
<td>P(vowel)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>“Go to Your Corners”</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problem Set 1</strong></td>
</tr>
<tr>
<td><strong>Question 3</strong></td>
</tr>
<tr>
<td>P(second half of the alphabet)</td>
</tr>
</tbody>
</table>
The Ultimate Cooperative Construction Challenge!

The Challenge: In groups of four, students are challenged to build a tower. The tower, made of straws, must be at least 50 cm tall and must have three small paper cups at the height of the tower. Candies will be used in the cups to test the strength of the structure. Other materials to be used could be plasticene, elastics, and paper clips. Each team has one pair of scissors. The time limit is set at 30 minutes.

The Extra Challenge: Each member in the group has a certain role to play.

1. One member of the group acts as the manager and is only allowed to speak! This group member is not allowed to touch any of the supplies.
2. The second member of the group is allowed to use only his/her right hand! The left hand is tied around the waist.
3. The third member of the group is allowed to use only his/her left hand! The right hand is tied around the waist.
4. The fourth member of the group is allowed to use both hands but is blindfolded!!!

This activity really imposes a structure on the group that focuses on cooperation for success! Watch and listen carefully as groups interact! This activity is fascinating!
If you're developing problem-based strategies in the classroom, promote this kind of thinking and attitude in the assignments you send home. Rethink the textbook and traditional worksheet approach. Practicing skills is important but learning comes from understanding and applying the concepts not necessarily from doing multiple examples of one type of problem. A worksheet can be a source of frustration for a student who didn't 'get' the concept and an aggravation for the student who has mastered the concept and doesn't need to prove it over and over again. Neither student will be highly motivated to complete this type of assignment!

Consider the following suggestions for homework assignments to promote mathematical thinking at home as well as in the classroom.

**The Worksheet Twist:**
Consider giving a traditional blackline master of multiple practice-type questions as an assignment but add a little twist. This activity works well with concepts, such as, order of operations, fractions, percents or integers. Explain to the students that they only have to complete five questions and you will give them the answers! Their challenge is to find the questions that produce the given answers. This twist adds several dimensions to the assignment. Firstly, it becomes a challenge - a game! The activity is motivating since students, being young adolescents, will try to do only five questions! Secondly, the activity engages estimation and number sense as
students survey the questions and mentally think of the possibilities. Students who have made connections will find the five questions fairly quickly. Students who need practice may very well have to do all the examples to complete the challenge but they will feel successful. Add a learning log entry asking students to explain their strategy in completing this challenge and you will gain an insight into their understanding of the concepts.

**Challenges that Promote Practice:**

Open-ended challenges where more than one solution is possible will encourage practice and success.

- **Consecutive digits:** Place operation signs between the digits nine through 1 as listed below to make an expression that equals 99. There are multiple solutions for this activity. Some solutions are found in 'Secret Ingredients', pg. 93.

  9 8 7 6 5 4 3 2 1 = 99

  example: 98 - 76 + 54 + 3 + 21 = 100

  This example would not be a valid solution but it shows how the digits stay in the same order.

- **Four Fours activity:** Use four fours, brackets and any operation sign to write an expression for each of the numbers 1 to 10. There may be more than one solution for each number. Some solutions are found in 'Secret Ingredients', pg. 93.

  Example: 44 - 44 = 0

- **Asking a non-traditional question** can engage students in problem solving! Challenge students to find out when they will be 100 000 days
old. Could a million dollars, of five-dollar bills, fit in a standard size suitcase?

- Use the digits 1 through 9 only once to make a true statement.
  Mathematicians claim there are 320 different solutions!

```
1 2 3
+ 4 5 6
---
7 8 9
```

**Connecting Problems to Real-life Situations:**

- When studying fractions, percents or ratio, ask students to investigate the percent or ratio of advertising space to information in one section of a newspaper. To extend this assignment, groups of students could be given different sections of the paper and report any differences or similarities. Ask further questions, such as, 'Does it make a difference if it's a weekend edition or a weekday paper?' or 'Will we find the same results in a magazine?' or 'Will different types of magazines have differing ratios?'

- Well before a special family event, such as, Thanksgiving or Christmas encourage students to get involved in family planning. Have students estimate how much a family get-together will cost by listing all that would be needed. Then encourage students to go shopping with their parents, record and compare actual prices.
• Challenge students to consider 'the ideal teenage bedroom' using the dimensions of their own bedroom as a starting point for this dream bedroom. Questions, such as, 'How many posters would it take to completely cover the walls of your bedroom?' will engage students in investigating area.

• While working on data management topics in class, consider a student assignment where keeping regular daily statistics is needed. Examples of topics could be precipitation rates, charting gas prices, charting the fluctuations of a stock portfolio, or keeping statistics on the performance of a sporting figure or team.

**Connecting Problems to the Classroom**

• For homework, have students develop practice questions or problems to give to their peers. Model solutions must be provided as well. Posing problems adds a sense of responsibility but also enables students to look at the structure of problems and what is needed. Will a peer be able to solve a problem if not enough information is given? What if assumptions are not clearly stated?

• Have students go on an information scavenger hunt as a homework assignment. If a topic is being discussed in class, have students find their own real-world connections by looking through newspapers or magazines for applications or by interviewing their parents or neighbours for suggestions of careers that incorporate this skill. Bringing these ideas to class and adding to other students' lists can help reinforce the theme that 'Math is all around us'!
A Balanced Diet

Final words

Many teachers, when asked if they incorporate problem solving into their daily math routine, respond by saying that they don’t have time to emphasize problem solving or their students don’t have the basic skills so they want to focus on drill! If our vision of Catholic educators is truly to help students become life-long, independent learners then a problems-based approach supports a ‘learning to learn’ attitude. The strategies and process of working through a challenge will remain with the student far longer than a specific skill or problem! A shift to problems-based instructional strategies will help students acquire the basic skills they need and will assist them to make meaningful and powerful connections.

A key element in successfully implementing this instructional philosophy will be your willingness to show your students that you are a curious and enthusiastic problem-solver! If you value and model the process, then your students will appreciate its value. Encourage students to persevere, explaining that skill in problem solving develops over time and through lots of practice! Celebrate small gains; highlight and share successes! Adding a little ‘sugar and spice’ is a recipe for a whole new flavour in your math classroom!
Sharing Recipes

Activity Sheets and Templates

Keep on Subtracting!
Template for Concept Mapping

What is it? (definition)

Properties/Facts:

Concept:

Comparisons:

Examples: (found in real life)
Template for Semantic Mapping

Concept:

Characteristic:

Example:

Characteristic:

Example:

Characteristic:

Example:

Characteristic:

Example:
# Frayer Template

<table>
<thead>
<tr>
<th><strong>Definition:</strong> Write a definition in your own words.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Characteristics:</strong> List attributes or qualities</td>
</tr>
<tr>
<td><strong>Examples:</strong> List examples. (What is it?)</td>
</tr>
<tr>
<td><strong>Non-examples:</strong> List non-examples. (What it isn’t!)</td>
</tr>
</tbody>
</table>

**Word or Concept:**

---

71
### K-N-W-S Template

<table>
<thead>
<tr>
<th><strong>K</strong></th>
<th><strong>N</strong></th>
<th><strong>W</strong></th>
<th><strong>S</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>What information do I know from reading the problem?</td>
<td>What information do I not need?</td>
<td>What am I asked to find? I will restate the problem in my own words.</td>
<td>What strategy (strategies) will I try to solve the problem?</td>
</tr>
</tbody>
</table>
### Five Step Process Chart

1. Write the problem in your own words.

2. What information do you need to solve the problem?

3. What strategies will you try? Plan your work.

4. Show your work. Explain using words, numbers and/or pictures.

5. Look back at your solution. Is your answer reasonable? Justify your reasoning.
Tiling

Card #1

Greatest common factor of 18 and 45

a multiple of 10

a prime number

a square number + 4

the prime factors of 30

x x x
Tiling

Card #2

\[
\begin{array}{c}
7 \\
\times 4 \\
\hline
2 \ 3 \\
\hline
3 \\
\hline
\end{array}
\]

\[
\begin{array}{c}
\underline{+3} \\
\hline
3 \\
\hline
6
\end{array}
\]
**Student Observation Chart: Learning Skills**

Excellent; Good; Satisfactory; Needs Improvement

<table>
<thead>
<tr>
<th>Names</th>
<th>Independent work</th>
<th>Initiative</th>
<th>Homework completion</th>
<th>Use of information</th>
<th>Cooperation</th>
<th>Conflict resolution</th>
<th>Participation</th>
<th>Problem Solving</th>
<th>Goal setting</th>
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</tbody>
</table>
**Student Observation Chart: Problem Solving**

<table>
<thead>
<tr>
<th>Names</th>
<th>Observations:</th>
<th>‘Wonder’ Phase</th>
<th>Working Phase</th>
<th>Review Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>
# Conference Recording

<table>
<thead>
<tr>
<th>Conference with:</th>
<th>Date:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focus of Discussion:</td>
<td></td>
</tr>
<tr>
<td><strong>Comments/Teacher</strong></td>
<td><strong>Comments/Student</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Conference with:</th>
<th>Date:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focus of Discussion:</td>
<td></td>
</tr>
<tr>
<td><strong>Comments/Teacher</strong></td>
<td><strong>Comments/Student</strong></td>
</tr>
</tbody>
</table>
**Thinking About My Problem Solving Approach**

Read the questions in each category. Place a 1, 2, 3, or 4 in each box to reflect on the process. Add specific comments to explain the number you chose.

<table>
<thead>
<tr>
<th>1 - I had no idea and needed instruction.</th>
<th>2 - I had difficulty and needed hints.</th>
<th>3 - I hesitated but I worked it out on my own.</th>
<th>4 - I knew what to do without hesitation.</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Reflect</th>
<th>Activity/Comments:</th>
</tr>
</thead>
<tbody>
<tr>
<td>'Wonder' Phase:</td>
<td>Did I list what I knew from the problem? Did I go get other information to help me? Did I think about any assumptions I was making? Did I think about how to organize the information? Did I restate the problem in my own words? Did I wonder about a strategy to use?</td>
</tr>
<tr>
<td>Working Phase:</td>
<td>Did I try a variety of strategies? Did I look for a pattern? If I was frustrated, did I go back and look at the problem? If I tried a strategy and it didn't work, did I try it again to see if the same would happen? Did I make errors in calculations?</td>
</tr>
<tr>
<td>Review Phase:</td>
<td>Does my solution make sense? Can I justify my solution? Did I communicate my solution effectively in writing? Did I look to see if there was a different solution?</td>
</tr>
</tbody>
</table>
### General Rubric: Categories in Mathematics Assessment

<table>
<thead>
<tr>
<th>Category</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concepts</td>
<td>Understanding of concepts: No / limited understanding of the concepts</td>
<td>Limited / adequate understanding of concepts</td>
<td>Understands concept</td>
<td>Deep understanding of concepts</td>
</tr>
<tr>
<td></td>
<td>Knowledge/skill level: Limited knowledge</td>
<td>Adequate knowledge</td>
<td>Satisfactory level of knowledge</td>
<td>High level of knowledge</td>
</tr>
<tr>
<td>Problem Solving/Making Connections</td>
<td>Understanding of problem: Needs assistance to understand problem</td>
<td>Partial understanding of problem</td>
<td>Needs no assistance to understand problem</td>
<td>Complete understanding of problem; plus insight</td>
</tr>
<tr>
<td></td>
<td>Planning: Needs assistance to select strategy</td>
<td>Needs some assistance to select strategy</td>
<td>Selects / develops effective strategies</td>
<td>Selects / develops / analyses highly effective strategies</td>
</tr>
<tr>
<td></td>
<td>Self-monitoring/reflection: No / little reflection on process or reasonableness</td>
<td>Reflects on process or reasonableness when asked</td>
<td>Reflects on process on reasonableness</td>
<td>Monitors progress &amp; changes strategies as needed</td>
</tr>
<tr>
<td></td>
<td>Risk-taking and perseverance: Seldom takes risks; on occasion perseveres when encouraged</td>
<td>Takes risks and perseveres when encouraged</td>
<td>Takes risks and perseveres</td>
<td>Enjoys taking risks, shows commitment and seeks challenges</td>
</tr>
<tr>
<td>Application</td>
<td>Connections: Has difficulty making connections</td>
<td>Needs encouragement to make connections</td>
<td>Applies concepts / skills in a variety of ways</td>
<td>Generalizes applications and explains relationships</td>
</tr>
<tr>
<td></td>
<td>Application of Skills: Inconsistent application of skills</td>
<td>Some inconsistency in application of skills</td>
<td>Proficient use of skills</td>
<td>Sophisticated / creative use of skills</td>
</tr>
<tr>
<td></td>
<td>Error Level: Work contains major flaws or many errors &amp; omissions</td>
<td>Work contains flaws or some errors &amp; omissions</td>
<td>Work may contain minor flaws</td>
<td>Work seldom flawed</td>
</tr>
<tr>
<td></td>
<td>Task Difficulty Level: Success on simple tasks</td>
<td>Success on tasks of moderate difficulty</td>
<td>Success on multi-step tasks</td>
<td>Success, even on non-routine tasks</td>
</tr>
<tr>
<td></td>
<td>Reading/listening skills: Requires assistance to get meaning from written / spoken input</td>
<td>Gets partial / adequate understanding from written / spoken input</td>
<td>Gets meaning from written / spoken input</td>
<td>Gets understanding and insight from written / spoken input</td>
</tr>
<tr>
<td></td>
<td>Questioning skills: Has difficulty asking clarifying questions</td>
<td>Needs encouragement to ask clarifying questions</td>
<td>Has no difficulty formulating clarifying questions</td>
<td>Asks ‘what if’, or extending questions</td>
</tr>
<tr>
<td></td>
<td>Explaining skills: No / incomplete explanation of answer / solution</td>
<td>Gives incomplete explanation of answer / solution</td>
<td>Gives complete explanation of answer / solution</td>
<td>Gives complete and compelling explanation of answer / solution</td>
</tr>
<tr>
<td></td>
<td>Conjectures/arguments: Makes unsupported statements, illogical conclusions</td>
<td>Supports conclusions with evidence when encouraged</td>
<td>Makes convincing arguments, makes / tests conjectures</td>
<td>Makes convincing / alternative arguments, insightful conjectures</td>
</tr>
<tr>
<td></td>
<td>Reporting skills: Requires assistance to communicate solution and results in writing, orally or in other forms of reporting</td>
<td>Communicate solution and results adequately in writing, orally or in other forms of reporting</td>
<td>Requires no assistance to communicate solution and results in writing, orally or in other forms of reporting</td>
<td>Communicates solution and results persuasively in all forms of communication</td>
</tr>
</tbody>
</table>

Cooperative Work Contract

We understand that to be successful in group work, each member of the group is responsible for a share of the project work. We intend to work, on task, on the following activities today:

<table>
<thead>
<tr>
<th>Group Member:</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

At the end of the class, reflect back on the task. Shade in how much of the task you feel you accomplished.

Signatures:

We were able to accomplish our goals today because:

We were frustrated in achieving our goals today because:

We will try the following strategies to be more successful:
Math Detectives – Sheet 1

Solve the following mysteries to crack the code.

A D E H M N P R S T U
33 24 10 2541 26 12 2450 23 28 6 48

Case 1: Number Suspects: 3, 4, 5, 6, 7, 8, 9, 10
Clue 1: The number is an even number.
The number is ____ The corresponding letter is ____.

Case 2: Number Suspects: 4, 6, 7, 11, 12, 13, 14, 16, 24
Clue 1: The number is less than 15.
The number is ____ The corresponding letter is ____.

Case 3: Number Suspects: 49, 143, 781, 942, 1012, 2541, 2450
Clue 1: The number is not a three-digit number.
The number is ____ The corresponding letter is ____.

Case 4: Number Suspects: 3, 4, 5, 6, 8, 12, 24, 28, 48
Clue 1: The number is less than 48.
The number is ____ The corresponding letter is ____.

Case 5: Number Suspects: 5, 6, 7, 8, 11, 13, 15
Clue 1: If you add 10 to the number, you get a number less than 20.
The number is ____ The corresponding letter is ____.

Case 6:
Clue 1: The number suspects are between 17 and 25.
Number Suspects: ____ ____ ____ ____ ____ ____
The number is ____ The corresponding letter is ____.

Case 7:
Clue 1: The number suspects are whole numbers less than 7.
Number Suspects: ____ ____ ____ ____ ____ ____
The number is ____ The corresponding letter is ____.

Case 8:
Clue 1: The number suspects are whole numbers between 26 and 34.
Number Suspects: ____ ____ ____ ____ ____ ____
The number is ____ The corresponding letter is ____.

Unscramble the letters to complete the phrase, 'Math is a study of ________'.
Math Detectives – Sheet 2

Solve the following mysteries to crack the code.

A D E H M N P R S T U
33 24 10 2541 26 12 2450 23 28 6 48

Case 1: Number Suspects: 3, 4, 5, 6, 7, 8, 9, 10
Clue 2: The number is not $4 \times 2$.
The number is _____. The corresponding letter is ____.

Case 2: Number Suspects: 4, 6, 7, 11, 12, 13, 14, 16, 24
Clue 2: The number is an even number.
The number is _____. The corresponding letter is ____.

Case 3: Number Suspects: 49, 143, 781, 942, 1012, 2541, 2450
Clue 2: The number is not $7 \times 7$.
The number is _____. The corresponding letter is ____.

Case 4: Number Suspects: 3, 4, 5, 6, 8, 12, 24, 28, 48
Clue 2: The number is greater than 10.
The number is _____. The corresponding letter is ____.

Case 5: Number Suspects: 5, 6, 7, 8, 11, 13, 15
Clue 2: If you subtract from the number, you get a number that is not zero.
The number is _____. The corresponding letter is ____.

Case 6: Number Suspects: ___ ___ ___ ___ ___ ___ ___
Clue 2: The number is an even number.
The number is _____. The corresponding letter is ____.

Case 7: Number Suspects: ___ ___ ___ ___ ___ ___ ___
Clue 2: The number is not an even number.
The number is _____. The corresponding letter is ____.

Case 8: Number Suspects: ___ ___ ___ ___ ___ ___ ___
Clue 2: The sum of the digits is greater than 5.
The number is _____. The corresponding letter is ____.

Unscramble the letters to complete the phrase, 'Math is a study of ________'.
Math Detectives – Sheet 3

Solve the following mysteries to crack the code.

A D E H M N P R S T U
33 24 10 2541 26 12 2450 23 28 6 48

Case 1: Number Suspects: 3, 4, 5, 6, 7, 8, 9, 10
Clue 3: The number is not $12 \div 3$.
The number is ____. The corresponding letter is ___.

Case 2: Number Suspects: 4, 6, 7, 11, 12, 13, 14, 16, 24
Clue 3: The number can be divided evenly by 3.
The number is ____. The corresponding letter is ___.

Case 3: Number Suspects: 49, 143, 781, 942, 1012, 2541, 2450
Clue 3: The number is not $2000 + 500 + 40 + 1$.
The number is ____. The corresponding letter is ___.

Case 4: Number Suspects: 3, 4, 5, 6, 8, 12, 24, 28, 48
Clue 3: The number is not two dozen.
The number is ____. The corresponding letter is ___.

Case 5: Number Suspects: 5, 6, 7, 8, 11, 13, 15
Clue 3: The number is not $5 + 2$.
The number is ____. The corresponding letter is ___.

Case 6:
Number Suspects: ____ ____ ____ ____ ____ __
Clue 3: The number is not $7 \times 3$.
The number is ____. The corresponding letter is ___.

Case 7:
Number Suspects: ____ ____ ____ ____ ____ __
Clue 3: The number is greater than 3.
The number is ____. The corresponding letter is ___.

Case 8:
Number Suspects: ____ ____ ____ ____ ____ __
Clue 3: The number 3 divides evenly into this number.
The number is ____. The corresponding letter is ___.

Unscramble the letters to complete the phrase, 'Math is a study of ________.'
Math Detectives – Sheet 4

Solve the following mysteries to crack the code.

<table>
<thead>
<tr>
<th>A</th>
<th>D</th>
<th>E</th>
<th>H</th>
<th>M</th>
<th>N</th>
<th>P</th>
<th>R</th>
<th>S</th>
<th>T</th>
<th>U</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>24</td>
<td>10</td>
<td>2541</td>
<td>26</td>
<td>12</td>
<td>2450</td>
<td>23</td>
<td>28</td>
<td>6</td>
<td>48</td>
</tr>
</tbody>
</table>

Case 1:  Number Suspects: 3, 4, 5, 6, 7, 8, 9, 10
Clue 4:  The number is greater than 8.
          The number is _____. The corresponding letter is ____.

Case 2:  Number Suspects: 4, 6, 7, 11, 12, 13, 14, 16, 24
Clue 4:  The number can also be divided evenly by 4.
          The number is _____. The corresponding letter is ____.

Case 3:  Number Suspects: 49, 143, 781, 942, 1012, 2541, 2450
Clue 4:  The sum of the digits is 11.
          The number is _____. The corresponding letter is ____.

Case 4:  Number Suspects: 3, 4, 5, 6, 8, 12, 24, 28, 48
Clue 4:  The number is not $3 \times 2 \times 2$.
          The number is _____. The corresponding letter is ____.

Case 5:  Number Suspects: 5, 6, 7, 8, 11, 13, 15
Clue 4:  The number is less than 7.
          The number is _____. The corresponding letter is ____.

Case 6:
Number Suspects:  ____  ____  ____  ____  ____  ____
Clue 4:  The number is not $10 + 9$.
          The number is _____. The corresponding letter is ____.

Case 7:
Number Suspects:  ____  ____  ____  ____  ____  ____
Clue 4:  The number can be divided evenly by 3.
          The number is _____. The corresponding letter is ____.

Case 8:
Number Suspects:  ____  ____  ____  ____  ____  ____
Clue 4:  The number is not $3 \times 9$.
          The number is _____. The corresponding letter is ____.

Unscramble the letters to complete the phrase, 'Math is a study of ________'. 
**Go To Your Corners**

**Problem Sets**

<table>
<thead>
<tr>
<th>“Go to Your Corners”</th>
<th>“Go to Your Corners”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem Set 2 Question 1</td>
<td>Problem Set 2 Question 2</td>
</tr>
<tr>
<td>Assign a probability to this event. The sun will rise tomorrow</td>
<td>Assign a probability to this event. The next baby to be born in the hospital will be a girl.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>“Go to Your Corners”</th>
<th>“Go to Your Corners”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem Set 2 Question 3</td>
<td>Problem Set 2 Question 4</td>
</tr>
<tr>
<td>Assign a probability to this event. You have type 'D' blood.</td>
<td>Assign a probability to this event. A card drawn from a standard deck will be a club. (Express as a %)</td>
</tr>
</tbody>
</table>

**Situation:** A card is drawn at random

<table>
<thead>
<tr>
<th>“Go to Your Corners”</th>
<th>“Go to Your Corners”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem Set 3 Question 1</td>
<td>Problem Set 3 Question 2</td>
</tr>
<tr>
<td>P(3)</td>
<td>P(prime)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>“Go to Your Corners”</th>
<th>“Go to Your Corners”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem Set 3 Question 3</td>
<td></td>
</tr>
<tr>
<td>P(shaded)</td>
<td></td>
</tr>
</tbody>
</table>
Secret Ingredients!

Solutions and Comments

Daily Specials!

Petals Around the Rose: Adding two odd numbers will result in an even total. The 'rose' is the dot in the middle. The 'petals' are the dot(s) around the rose. To get the total you count up all the petals! It is such an easy pattern - if you see it!

Snacks: Bet you can’t serve one!
Students will want more!

Challenge: Can you walk through a piece of paper?

Fold the paper in half along its length and cut as shown. Flatten out the paper and cut the center piece except the border on each end. If you use a piece of card stock and make your cuts narrow, you can create a loop that even the tallest person in the room can walk through!!!
Appetizers:

Warm-up Activities

Tiling 1

Greatest common factor of 18 and 45

\[
9
\]

A prime number

\[
7 \quad 1
\]

The prime factors of 30

\[
2 \times 3 \times 5
\]

Tiling 2

A multiple of 10

\[
\begin{array}{c}
7 \\
9 \\
3
\end{array}
\]

A square number + 4

\[
\begin{array}{c}
2 \\
3 \\
7
\end{array}
\]

+ \[
\begin{array}{c}
3 \\
1 \\
6 \\
8
\end{array}
\]

3. 1, 5, 3 in the corner circles
2, 4, 6 to fit appropriately
If the total was 12: 6, 5, 4 in corners

4. 4 possibilities

\[
19 + 7 + 3 = 29; 7 + 7 + 15 = 29; 11 + 11 + 7 = 29; 15 + 11 + 3 = 29
\]

5. Brad earned $8, $10, $12, $14 and $16.

6. The letter 'A' could be 3, 6, or 9

7. Siying is now 33 years old; Siu is 11 years old.
8. Answers may vary. The 5 is always in the center box and the corner boxes are even numbers.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>8</td>
</tr>
</tbody>
</table>

9. 2 tables and 20 stools;
5 tables and 16 stools;
8 tables and 12 stools;
11 tables and 8 stools;
14 tables and 4 stools;
17 tables and no stools. This possibility should be eliminated since the question stated that the furniture store did have both tables and stools.

10. The room is 6 m x 3 m. A square, 4 x 4, has an area of 16 m² and a perimeter of 16 m.

11. This is a great activity to get students to think 'outside the box.'

12.
13.

14. Two weighings. Place three coins in one pan and three coins in the second pan. If the pans balance, then the counterfeit is one of the remaining three. If the pans do not balance, the counterfeit is in the pan that is higher. In either case, you have reduced the search to three coins. From these three coins, place one coin in one pan and a second coin in the second pan. If the pans balance, then the third coin is the counterfeit. If the pans do not balance then the coin in the higher pan is the counterfeit.

If you knew that the counterfeit was heavier you would be looking for the lower pan.

If you only knew that the counterfeit was a different weight then more weighings would be necessary.

15. Jennifer, Jalal, Melanie, Judy, Mark

16. 28 games. Each person must play every other person once. The chart below organizes the games. Note: - denotes that this player has already played one game against the second person.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>x</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>B</td>
<td>-</td>
<td>x</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>C</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>D</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
</tr>
<tr>
<td>E</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>23</td>
<td>24</td>
<td>25</td>
</tr>
<tr>
<td>F</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>26</td>
<td>27</td>
</tr>
<tr>
<td>G</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>28</td>
</tr>
<tr>
<td>H</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>x</td>
</tr>
</tbody>
</table>
17. Fewest: 5 pieces; Most: 11 pieces

18.

<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickels</th>
<th>Pennies</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>23</td>
</tr>
</tbody>
</table>

19. Lance bought 24 doughnuts. Students have difficulty using the strategy 'working backwards'. This is a good question to highlight this strategy.

20. 15 numbers: (1, 3, 5, 7, 9), (11, 13, 15, 17, 19), (31, 33, 35, 37, 39)

Smorgasbord!

Marshmallow – Toothpick Challenge: It doesn’t really matter how many toothpicks and marshmallows you give the students! A series of linked tetrahedrons will allow the structure to hold the videocassette. Many students will wonder how the vertices of the tetrahedrons can support the videocassette. Discussions based on the strength of this structure can be linked to Science expectations.

Palindromic Reversals: It will take 24 reversals to create the palindrome 8813200023188! A great activity to practice addition requiring patience and perseverance!
Calendar Challenge:
A. The sum of the diagonals will be equal.
B. The sum will always be the center number.

<table>
<thead>
<tr>
<th></th>
<th>X - 8</th>
<th>X - 7</th>
<th>X - 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>X - 1</td>
<td>X</td>
<td>X +1</td>
<td></td>
</tr>
<tr>
<td>X +6</td>
<td>X +7</td>
<td>X +8</td>
<td></td>
</tr>
</tbody>
</table>

Jelly Bean Jars:
Constructing a table is one way to solve the problem.

<table>
<thead>
<tr>
<th></th>
<th>Guess</th>
<th>-4.5</th>
<th>-2.2</th>
<th>-1.3</th>
<th>+4.5</th>
<th>+2.2</th>
<th>+1.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maria</td>
<td>4.7</td>
<td>0.2</td>
<td>2.5</td>
<td>3.4</td>
<td>9.2</td>
<td>6.9</td>
<td>6.0</td>
</tr>
<tr>
<td>Jason</td>
<td>8.2</td>
<td>3.7</td>
<td>6.0</td>
<td>6.9</td>
<td>12.7</td>
<td>10.4</td>
<td>9.5</td>
</tr>
<tr>
<td>Kyle</td>
<td>11.4</td>
<td>6.9</td>
<td>9.2</td>
<td>10.1</td>
<td>15.9</td>
<td>13.6</td>
<td>12.7</td>
</tr>
</tbody>
</table>

The jar of jelly beans must weigh 6.9 kg since it is the only number that appears in each row.

Partition of a Square:
Divide the length or width into 6 cm and 4 cm and the other side into 3 cm by 7 cm.
Equivalent Fractions:
3/6, 9/18, 27/54

Investigate Powers:
Except for the first two examples, the expression with the greater exponent will be the greater number. That is \( n^{n+1} \) will be greater than \((n+1)^n\) for all positive integers where \( n \) is greater than 2.

Movie Promotion:
The twenty-fourth, the forty-eighth and the seventy-second student passing through the admission gate received both hat and a T-shirt. The first three common multiples of 8 and 12 are, 24, 48 and 72.

Pass the Cookies:
35 ways: four ways to get 1-1-1-5; twelve ways to get 1-1-2-4; six ways to get 1-1-3-3; twelve ways to get 1-2-2-3; one way to get 2-2-2-2.

Number a Cube:

![Diagram of a cube]

Tip the Driver:
The driver received $8.41 in equal amounts of nine coins from all riders. Factoring 841 leads to \( 29 \times 29 \) (29 cents \( \times \) 29 riders). The only way to obtain 29 cents with nine coins is 5 nickels and 4 pennies. Therefore, the driver did not get any dimes! Students will groan at this solution but it's a good problem to lead into a discussion about assumptions.

Pizza Pizza:
The waiter was correct. The ratio of the area is 5024 cm\(^2\) to 1962.5 cm\(^2\) (40 cm diameter to 25 cm diameter) which is greater than a 2:1 ratio.
Red, Green and Blue:
Twenty-four green marbles must be added. If green marbles must represent $3/4$, then nine red and blue marbles must represent $1/4$. If 9 represents $1/4$ then 27 represents $3/4$. To get 27 from 3, 24 green marbles must be added.

I’ll Wash! You Dry!

Cooperative Learning in Math

Go To Your Corners

Problem Set 1:
1) $P(\sim E)=10/11$  
2) $P(\text{vowel}) = 4/11$  
3) $P(\text{second half of alphabet}) = 2/11$

Problem Set 2:
1) $P = 1$  
2) $P = 50\%$  
3) $P = 0$  
4) $P = 25\%$

Problem Set 3:
1) $P(3) = 1/10$ or $10\%$  
2) $P(\text{prime}) = 4/10$ or $40\%$  
3) $P(\text{shaded}) = 5/10$ or $50\%$

Fast Food

Taking Math Home

Challenges that Promote Practice:
Consecutive digits: Other solutions are possible
$9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 99$

Four Fours activity: Other solutions are possible
$44 \div 44 = 1$  
$(4 + 4 + 4) \div 4 = 3$  
$(4 \times 4 + 4 ) \div 4 = 5$  
$4 + 4 - (4 \div 4) = 7$  
$4 + 4 + (4 \div 4) = 9$

$(4 \times 4) \div (4 + 4) = 2$  
$4 + 4 (4 - 4) = 4$  
$4 + (4 + 4) \div 4 = 6$  
$4 + 4 + 4 - 4 = 8$  
$(44 - 4) \div 4 = 10$
Food for Thought!  

Text Resources; Web Sites

Text Resources:


Onslow, B. Sauer, R. Ed. (2001). *Linking Assessment and Instruction in Mathematics: The Middle Years.* OAME.


**Magazines**


**Web Sites:**

http://www.funbrain.com

  Math games for all ages.

http://www.eduplace.com/math/brain

  Brain Teasers: Primary/Junior/Grades 7 & 8

http://www.mathgoodies.com/lessons/

  Lesson plans for teachers; activity worksheets
http://stfx.ca/special/mathproblems/
Sponsored by Canada’s School Net. Math problems for grades 5 to 12.

http://mathforum.com//dr.math
Elementary Problem of the Week; Middle School Problem of the Week; Geometry Problem of the Week; Algebra Problem of the Week

http://www.mathforum.com//mathmagic
Problem of the Week- Grades 7 - 9

A good site for problem solving by asking uncommon questions and encouraging students to think mathematically.

http://www.musemag.com/
Muse Magazine website with links to 'Cool Math Stuff'.

www.ex.ac.uk/cimt
Site of Centre For Innovation in the Teaching of Mathematics. Puzzles, challenges and activities for students and teachers.

www.maa.org/mathland/mathland_archives.html
Ivars Petersen’s Mathtrek and Mathland links to home page of Mathematics Association of America.

www.nctm.org
National Council for Teachers of Mathematics sets standards for states and provinces.

http://mathcentral.uregina.ca/
The site of Math Central - University of Regina. Try the Resource Room and Questions and Quandries.

http://juliet.stfx.ca/people/stu/x94emj/teacher.htm
This is the Canadian School Math page from St. Francis Xavier University. Visit the Elementary Math page with links to Berritts Best Sites For Kids.