An Annotated Learning Journey within an Intermediate Classroom

Setting the Context: Grade 7/8

Ontario Catholic School Graduate Expectations: CGE 2b, 3c, 4b, 5a, 7b

Gospel Values/Virtues: Justice, Excellence, Hope, Community, Love

Learning Skills and Work Habits: Collaboration, Initiative, Organization, Independent work, Self-Regulation

Curricular Learning Goals:
I can compare experimental probabilities with the theoretical probability of an outcome (Grade 7).
I can represent in a variety of ways, all the possible outcomes of a probability experiment (Grade 7).
I can use probability models to make predictions about real-life events (Grade 8).
I can compare the theoretical probability of an event with the experimental probability and explain why they differ (Grade 8).

The educator wonders...
"How might this experience strengthen each child's own self-efficacy toward mathematical thinking and establish a strong Catholic learning community?
What is the connection between fairness and justice? How might I apply this to Catholic Social Teaching?
What are the learning goals?
What kind of a learning task will reach each student's needs?
How might this task allow students to uncover big ideas from the curriculum and relate these to real-world contexts?
How might this task be designed to be engaging for all students?
How much time would be devoted to this learning experience?"

Revealing the educator's thinking:
"From past experiences and in discussion with colleagues at the school, I know students will enter this learning experience with some familiarity with experimental and theoretical probability. During the activation I will observe students in their understanding of tree diagrams and fraction multiplication to determine theoretical probability."

"Will they use any other strategies? Will I be able to uncover any misconceptions?"
"I value the processes of reasoning, proving, reflecting and representing thinking. How might I convey the importance of process and downplay the 'right' answer?"

Planning with the end in mind:
The educator has a plan for this learning that will lead students through a series of experiences from simple to more complex in order to allow students to construct meaning aligned with the learning goals. Anchoring the learning on the big idea of justice and fairness engages the students in a conversation that is relevant for their developmental stage of thinking.

The learning journey involves more than what goes on in the classroom. Collaboration between educators, whether with same grade partners in the school, or system-wide, means there are many educators involved in the learning journey. This is an example of **Solidarity**. It also prepares educators (and students to always consider **the Common Good.**

Revealing the educator's intentional decisions to allow for student-centred collaborative inquiry:

- "During the activation, students were asked to predict which version was fair. Most students assumed version 2 would be fair. Students were surprised when determining the theoretical probability and concluding that the third version was actually fair."
- "We looked at representations of student thinking. Most students used tree diagrams. Although effective in a simple example, I knew that during the **Working on it** phase students would also need fraction multiplication. Using samples of student work, I modelled this strategy for one version and then we worked together through shared practice to apply the same strategy to the two other versions."
- "While introducing the more challenging task, I encouraged students to persevere, to look back at their previous work for inspiration, to view incorrect attempts as opportunities to make adjustments that will bring them closer to the correct combination."
- "I intentionally paired students of like ability who I knew would be able to persist together."

Assessment *for, as* and *of* Learning:
The educator is continuously assessing while the students are engaged in their inquiry. There is a moral imperative to allow purposeful talk and observation of student interaction to guide instruction for each student on an individual level. The educator will use effective questioning to provide feedback and provoke student thinking. In this phase of learning, the feedback is timely, meaningful and provides opportunities for assessment *as* learning. Student reflect on their own choices and adjustments to provide direction on the purpose and intentionality of the task.

The Learning Experience:

All's Fair:
What is fairness?
What does it take for a game to be considered fair?

This learning experience should take three 80-minute blocks.

The Instructional Tasks:
These open-routed tasks were chosen to foster the process of 'solution finding'. Students discover what is considered fair within the simpler activation task. Although they know there is one correct solution to the more complex task, they also know there are many ways to attack the problem. They are challenged to take risks, to make errors and analyze these in order to pinpoint a solution through their attempts. The element of choice is an essential element of differentiated instruction allowing the dignity of the learner to be respected as it values the child's learning journey.

Students are encouraged to collaborate, share ideas and give advice to each other. This promotes a culture of community, respecting each other's thinking and fosters mathematical discussion.

The concept of fairness in a game brings rich dialogue to the classroom, "What constitutes fairness—even when we are playing games? What is **justice** in a context of Catholic Social Teaching?" Living in *right relationship* with God and our neighbour calls us to love kindness and to walk humbly (Micah 6:8) How might instructional tasks be designed to promote this Catholic teaching?

Activation:

What does it take for a game to be considered fair?
Through discussion the class concluded that a fair game is one in which all players have the same chance of winning. In the case of a two-player game, each player would need to have a 50% chance of winning.

The students were introduced to a simple game called ***All's Fair*.*** In this game, two tiles are drawn from a bag. If the tiles are the same colour, Player 1 gets a point. If the tiles are different colours, Player 2 gets a point.

Students were asked to consider three versions of All's Fair: 1) the bag contains 1 blue tile and 2 red tiles; 2) the bag contains 2 blue tiles and 2 red tiles; 3) the bag contains 1 blue tile and 3 red tiles.

The task was to determine using theoretical probability, which of the three versions, if any, was fair.

How does the activation phase support the learning?
The activation phase provokes thinking and leads into the more complex learning experience. In this problem, the educator is uncovering background knowledge and listening for any misconceptions. This is important assessment *for* learning information which will allow him/her to support students as they move to the more complex task.

Allowing students to hear each others' thinking generates ideas, validates a student's thinking and creates a safe, respectful environment for a student to question a peer or herself/himself. This is important for all students, but necessary for students who may struggle. Everyone should feel eager and ready to tackle the more complex problem after the activation. This allows the child's innate sense of wonder and awe in discovery to manifest itself.

Working on it:

What if you wanted to create a fair version of *All's Fair that used three different colours of tiles? How many tiles of each colour would be needed?**

The class referred back to the work in the Activation section and established the similarities and differences between it and the new task. The criteria below was co-constructed:

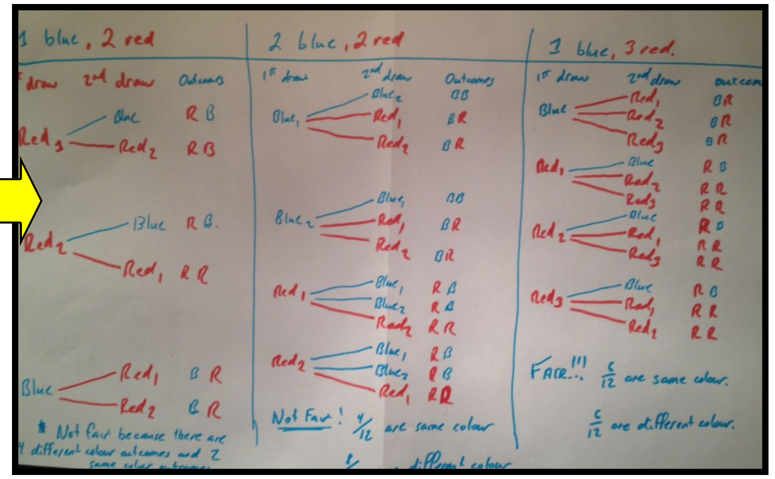
- Working with a partner, decide how many tiles of each colour you would put in the bag. Use up to 10 tiles of each colour.
- Determine the experimental probability of the game by conducting at least 20 trials. Record the results.
- If you feel the experimental results do not reflect a fair game, change the numbers and try again.
- If the results of the experiment are close to a fair game, use an appropriate method to determine the theoretical probability of that combination of tiles.
- Student thinking should be represented throughout the process. "Once you have a version that is fair, or as close to fair as you think you'll be able to get, finalize your thinking on chart."

*Adapted from: (1998). 'Even Steven', *The Super Source: Probability and Statistics: Grades 7 and 8.* ETA/Cuisinaire: White Plains, New York, p.17.

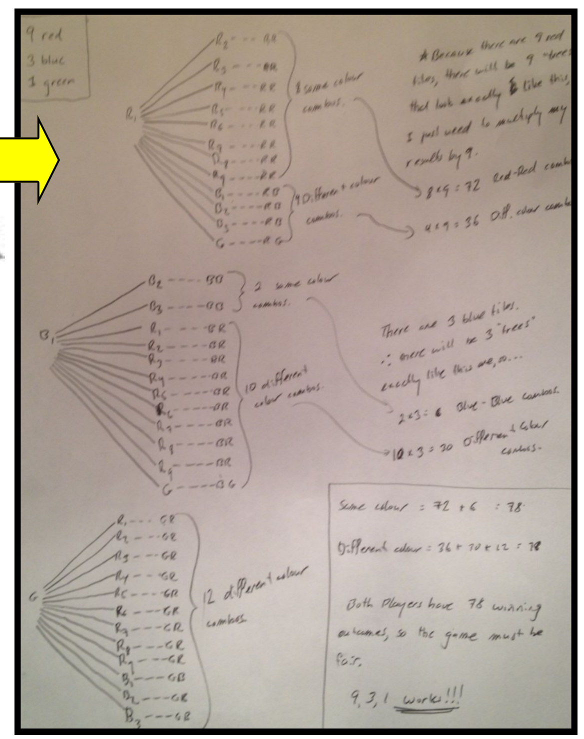
A Glimpse of Student Thinking:

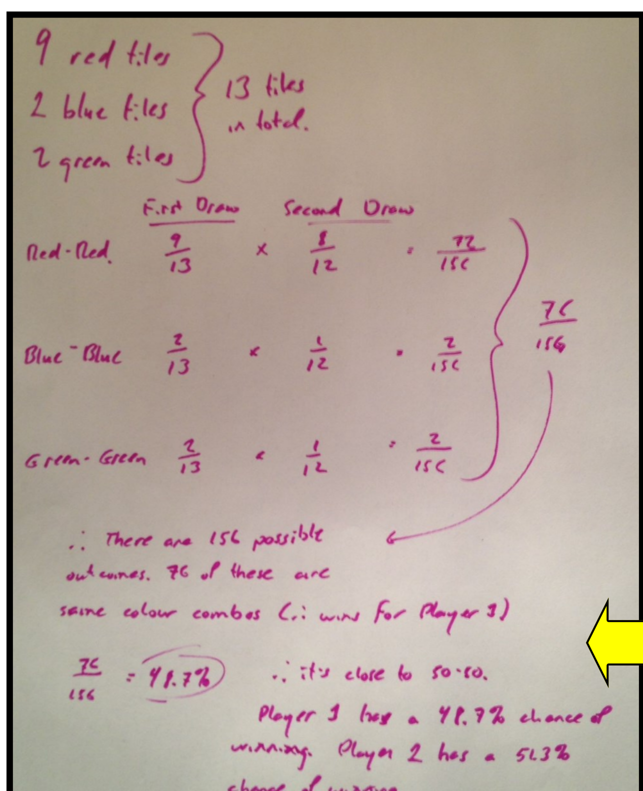
To honour the thinking of each student, it is imperative that they be given multiple opportunities to demonstrate their learning in a variety of ways in order to show growth over time. Observations, conversations and written products, aligned to learning goals and success criteria, are gathered and documented by educators and students.

Sample 1: This sample depicts the solutions for the three versions presented in the activation. This student-generated work is posted and used as an anchor chart to guide the students during the **Working on it** phase.



Sample 2: Students realized that all tree diagrams starting with the same colour tile were the same. They saved time by building one tree diagram for each colour of tile for the first draw, then multiplied their results by the number of tiles of that colour in the bag. In this way, they were able to find an efficient and reliable strategy.





Sample 3: Process is more important than a correct solution. In this sample, the students, after many efforts, narrowed their thinking down to a combination that yields results that are so close to being fair that no one playing the game would be able to tell that one player had a slight advantage. Is this fair?

Consolidation:

Although a correct solution was not required for the ‘success’ of this learning experience, the consolidation phase was important as a way to celebrate the thinking of each student, to gain insights about the use of strategies and to learn from errors or misconceptions.

A gallery walk was organized by posting student work around the room. As a whole the co-constructed criteria class was reviewed. The process of giving specific, descriptive and appropriate feedback aligned to the criteria was modelled.

Students read each of the work samples carefully and used post-it notes to provide meaningful feedback. At the end of the gallery walk, each pair returned to their own chart and read the feedback provided by their peers. Students were given time to respond to the feedback by adding or refining their work.

As students were refining their work, evidence of learning was documented through individual conferencing. Students were encouraged to take photos of their work and explain their thinking about what they learned.

During the whole class debrief students reviewed, named and prioritized strategies making connections to the learning goals. Several charts were annotated with specific criteria to use as anchor charts to guide further work on probability.

The use of a galley walk format for consolidation is one way to foster the sharing of each student’s thinking. When a culture of willingness to share is established, students are eager to see what other students have done. There is no judgement in this action. It is an act of acceptance and respect.

Intentionally modelling how to give specific, descriptive feedback aligned to learning goals and success criteria is essential to building a climate of respect and support.

Allowing students to respond to feedback by creating structures and giving time to the process, validates the worth of each student’s thinking.

Giving time for students to adjust and reflect on thinking gives the educator time to meet with each student individually and use this conversation, coupled with the student work, as assessment *of* learning.

Reflection:

The process of two students: “One partnership (DJ and SM) started with one combination of numbers, determined the theoretical probability, and then adjusted only one of their numbers by a single digit. If that change brought them closer to a 50-50 split, they continued to change their numbers in that direction. If it did not, then they returned to their previous stage and tried a different change. After many attempts, they had found numbers that gave Player 1 a 49% chance of winning and Player 2 a 51% chance of winning. They had only one small adjustment to make. They made several changes, many of which actually moved them further away from a fair game. I heard a triumphant roar. There stood DJ with arms raised like he had just won the Stanley Cup. Pure joy of learning!”

The educator’s process: During the working phase, the educator reflects on how he might use effective questioning to move the thinking forward. For students who are struggling, “How might the results of the activation inform this more complex problem?” Talking through what students know (i.e. a combination of 1 red tile and 3 blue tiles proved to be fair combination).

Another way to ‘push’ the thinking of students is to ask, “How would knowing the total number of tiles used, help to limit the possible number combinations?”

Some students will need a greater challenge and will be asked to consider the working combinations for 4 colours, 5 colours, etc.

What does the joy of learning look like and sound like? The learning journey of these two students epitomizes the spirit of engagement and the power of joy when adversity has been conquered. Success is in the journey, not necessarily in the end destination. Asking students what they learned—not just the mathematical concepts, but as a learning community, would strengthen their mindset of success.

The educator considers opportunities for individual accountability to serve as assessment *of* learning. Possible next steps include: choice of responding to a similar question or developing a fair game that is based on probability using simple materials and explaining why their game is fair.

Next Steps:

“This is a very challenging task that requires a lot of fortitude and perseverance. I have seen many very capable students not get the final answer. However, I believe that solving the problem is not of primary importance. If students make predictions, determine probabilities, reflect on their results and use them to adapt their predictions in logical ways, then I feel they have succeeded. There is so much rich mathematical thinking involved before they get to a solution that the solution itself becomes unimportant.”

“I want students to make connections to fairness and justice in the world—in everything they do. I wonder how to build on this concept in all subject areas and relate their mathematical learning to relevant experiences in their own life?”

How might our faith teach us about justice? Which principle of Catholic Social Teaching might we investigate to strengthen our understanding of justice?

- Dignity of the Human Person
- The Common Good
- Solidarity
- Subsidiarity
- Preferential Option for the Poor
- Dignity of the Worker
- Peace
- Universal distribution of goods
- Environmental stewardship

An Annotated Learning Journey within an Intermediate Classroom

Contextual Information

Overall Expectations:
Grade 7: Compare experimental probabilities with the theoretical probability of an outcome.
Grade 8: Use probability models to make predictions about real-life events.

Specific Expectations: Grade 7:
- Represent in a variety of ways, all the possible outcomes of a probability experiment involving two independent events, and determine the theoretical probability of a specific outcome involving two independent events;
- Perform a simple probability experiment involving two independent events, and compare the experimental probability with the theoretical probability of a specific outcome.

Specific Expectations: Grade 8:
- Compare, through investigation, the theoretical probability of an event (i.e., the ratio of the number of ways a favourable outcome can occur compared to the total number of possible outcomes) with experimental probability, and explain why they might differ;
- Determine, through investigation, the tendency of experimental probability to approach theoretical probability as the number of trials in an experiment increases, using class-generated data and technology-based simulation models;
- Identify the complementary event for a given event, and calculate the theoretical probability that a given event will not occur.

Connecting the Mathematical Processes to Curricular Goals:
Reasoning and Proving: develop and apply reasoning skills to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;

Reflecting: demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem;

Connecting: make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts;

Representing: create a variety of representations of mathematical ideas, make connections and compare them, and select and apply the appropriate representations to solve problems;

Communicating: communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

Developing Ontario Catholic School Graduate Expectations:
An Effective Communicator who:
CGE2b Reads, understands and uses written materials effectively.

A Reflective and Creative Thinker who:
CGE3c Thinks reflectively and creatively to evaluate situations and solve problems.

A Self-Directed, Responsible, Lifelong Learner who:
CGE4b Demonstrates flexibility and adaptability.

A Collaborative Contributor who:
CGE5a Works effectively as an interdependent team member.

A Responsible Citizen:
CGE 7b Accepts accountability for one's own actions.

The Effective Use of Manipulatives

The key to the successful use of manipulatives lies in the bridge – which must be built by the teacher – between the artifact and the underlying mathematical concepts. The mathematics is in the connections, not the objects.

Ontario Ministry of Education. (2004), *Teaching and Learning Mathematics: The Report of the Expert Panel on Mathematics in Grades 4 to 6 in Ontario,* p. 61.

***The Effective Use of Manipulatives:**
"Students will investigate mathematical concepts using a variety of tools and strategies, both manual and technological. Manipulatives are necessary tools for supporting the effective learning of mathematics by all students. These concrete learning tools invite students to explore and represent abstract mathematical ideas in varied, concrete, tactile, and visually rich ways. Moreover, using a variety of manipulatives helps deepen and extend students' understanding of mathematical concepts. Manipulatives are also a valuable aid to teachers. By analyzing students' concrete representations of mathematical concepts and listening carefully to their reasoning, teachers can gain useful insights into students' thinking and provide supports to help enhance their thinking."

"When selecting manipulatives, teachers should:
- be certain that the manipulatives chosen support the selected mathematics concepts and big idea;
- have enough of the manipulative available so that all students can become active participants in the activity;
- provide initial opportunities for students to become familiar with the manipulative;
- communicate classroom procedures to students (e.g., refrain from using the manipulative when someone is sharing with the class; put manipulatives away in containers)."

"When planning activities with manipulatives, teachers should:
- use a manipulative in such a way that students then use it as a "thinking tool" enabling them to think about and reflect on new ideas;
- recognize that individual students may use the manipulative in different ways to explore mathematics;
- avoid activities that simply ask children to copy the actions of the teacher;
- allow students to use manipulatives to justify their solution as well as solve the problem;
- take time to become familiar with the manipulative chosen;
- choose manipulatives that will allow students to represent the mathematics in a meaningful way and to make connections from these representations;
- provide opportunities for students to explore the same concept with a variety of manipulatives."

* Ontario Ministry of Education. (2003). *Early Math Strategy: The Report of the Expert Panel on Early Mathematics in Ontario.* pg.21–24.

Effective Feedback

"I have struggled to understand the concept of feedback. The mistake I was making was seeing feedback as something teachers provided to students...it was only when I discovered that feedback was most powerful when it was from the student to the teacher that I started to understand it better. When teachers seek, or at least are open to, feedback from students as to what students know; what they understand, where they make errors, when they have misconceptions, when they are not engaged—then teaching and learning can be synchronized and powerful. Feedback to teachers helps make learning visible."

John Hattie. (2009). *Visible Learning.* p. 173.

**"Feedback can take many forms. Instead of saying, 'This is what you did wrong' or 'This is what you need to do,' we can ask questions: 'What do you think you need to do? What other strategy choices could you make? Have you thought about . . .?'"*

***Using Assessment to Promote Learning: Providing Feedback**

"When students experience difficulties and receive no useful feedback, they are likely to attribute their problems to a lack of ability and give up. On the other hand, when students receive specific information about ways in which they can improve, and are given opportunities to revise their work, they have a clear message from the teacher that they are capable of learning and improving.

Quality feedback goes beyond praise – it informs the student about successes, areas for improvement, and next steps to extend learning.

Effective feedback helps students:

- recognize what they have done well, and how they might improve;
- overcome obstacles by building understanding on what students already know;
- set improvement goals."

"Setting Clear Criteria

As teachers are preparing students for learning activities, they need to communicate clearly to students what is expected of them. Without clear guidelines, students might produce work that is not indicative of what they actually know or can do."

"Demonstrating Characteristics of Quality Work

Students should know what quality work looks and sounds like.

- Teachers play an important role in modelling processes for learning mathematics: how to use materials to represent mathematical ideas, how to perform mathematical procedures, how to solve problems, how to communicate mathematical ideas orally and in writing. As teachers demonstrate these processes, they talk aloud, making their thoughts explicit to the students. Often, through guided mathematics experiences in which students and teacher are working collaboratively to solve a problem, investigate a concept, or write mathematical ideas, the teacher can focus students' attention on behaviours and characteristics that constitute quality work.
- Teachers and students share examples of quality work. Teachers can ask a student to show and talk about his or her work when some aspect of the work would help other students to improve.
- Teachers can explain characteristics of quality work to students using simple rubrics, classroom displays, and checklists. Teachers can discuss the characteristics of good work with students before, during, and after learning activities in order to clarify what students should strive for."

*Ontario Ministry of Education. *A Guide to Effective Instruction in Mathematics: Kindergarten to Grade 6—Volume 4: Assessment and Home Connections.* pg. 16-17.

Ways to Consolidate Learning

***Developing Effective Mathematical Communication:**

The development of students' mathematical communication shifts in precision and sophistication throughout the primary, junior and intermediate grades, yet the underlying characteristics remain applicable across all grades. During whole-class discussion, teachers can use these characteristics as a guide both for interpreting and assessing students' presentations of their mathematical thinking and for determining discussion points.

The characteristics, listed below, are relevant across grades:

- **precision** about problem details, relevant choice of method or strategy to solve the problem, accurate calculations;
- **assumptions and generalizations** that show how the details of the mathematical task/problem are addressed in the solution;
- **clarity** in terms of logical organization for the reader's ease of comprehension, requiring little or no reader inference;
- a **cohesive argument** that consists of an interplay of explanations, diagrams, graphs, tables and mathematical examples;
- **elaborations** that explain and justify mathematical ideas and strategies with sufficient and significant mathematical detail;
- appropriate and accurate use of **mathematical terminology, symbolic notation and standard forms** for labelling graphs and diagrams."

"Through listening, talking and writing about mathematics, students are prompted to organize, re-organize and consolidate their mathematical thinking and understanding, as well as analyze, evaluate and build on the mathematical thinking and strategies of others. The use of mathematical language helps students gain insights into their own thinking and develop and express their mathematical ideas and strategies, precisely and coherently, to themselves and to others. It is during whole-class discussion that students explain and justify their ideas and strategies as well as challenge and ask for clarification from their classmates."

"Gallery Walk is an interactive discussion technique that gets students out of their chairs and into a mode of focused and active engagement with other students' mathematical ideas (Fosnot & Dolk, 2002). The purpose of the Gallery Walk is to have students and the teacher mathematically engage with a range of solutions through analysis and response.

For students, Gallery Walk is a chance to read different solutions and provide oral and written feedback to improve the clarity and precision of a solution. On the other hand, for teachers, it is a chance to determine the range of mathematics evident in the different solutions and to hear students' responses to their classmate's mathematical thinking. Such assessment for learning data help the teacher to determine points of emphasis, elaboration and clarification for the ensuing whole class discussion (Fosnot & Dolk, 2002)."

"Math Congress is a mathematics instructional strategy developed by Fosnot and Dolk (2002). The purpose of the congress is to support the development of mathematicians in the classroom learning community. A congress enables the teacher to focus the students on reasoning about a few big mathematical ideas derived from the mathematical thinking present in the students' solutions. A Math Congress is not about showing every solution, as there is not enough time, nor is every student at the same place where the strategy will make sense to them. Instead, it focuses whole-class discussion on two or three, strategically selected, student solutions in order to develop every student's mathematical learning."

"Bansho (Board Writing): The purpose of Bansho is to organize and record mathematical thinking derived from and collectively produced by students on a large-size chalkboard or dry erase board. Because this written record enables simultaneous comparison of multiple-solution methods, there is the potential for students to construct new mathematical ideas and deepen their mathematical understanding."

*Ontario Ministry of Education. (Sept. 2010). *Communication in the Mathematics Classroom.* pg. 1– 4.